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Research Article

Multicriteria Adaptive Observers for Singular Systems with Unknown Time-Varying Parameters

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This paper proposes multicriteria adaptive observers for a class of singular systems with unknown time-varying parameters. Two criteria for the H_∞ disturbance attenuation level and the upper bound of an ultimate invariant set are scalarized into a single cost function and then it is minimized by varying the weight parameter, which creates the optimal trade-off curve or Pareto optimal points. The proposed multicriteria adaptive observers are shown to be able to easily include integral action for better robust performance. It is demonstrated with numerical simulations that the proposed multicriteria adaptive observers provide the good estimation accuracy and allow effective and compromising design by considering two different cost functions simultaneously.

1. Introduction

State estimation, or observation, has been recognized as one of the important research issues for dynamic feedback control systems since the full state information required for high performance is not available in most cases due to the high cost of sensors and limited accessibility for measurement. For state estimation, various types of observers have been developed, including Luenberger observers [1], sliding mode observers [2], and robust observers [3].

In the presence of unknown parameters encountered in most real systems, the observers designed for nominal models are hard to be applied in practical applications. For this reason, adaptive observers have been developed to estimate unknown parameters as well as state variables from input and output measurements, and hence achieve the robustness [4–8]. Recently, the results on adaptive observers have been successfully extended even to more general singular systems [9, 10]. Singular systems have extensive applications in many practical systems such as electrical systems, economics, mechanics, and chemical processes.

In implementing such practical adaptive observers over general singular systems, several criteria can be taken into account in consideration of design specifications. For example, adaptive observers can be designed according to the criteria such as H_∞ [11, 12], H_2, the ultimate region size, and so on. Mostly, among them, only one criterion has been employed for design of adaptive observers. However, two or more criteria could be applied to involve multiple design objectives, leading to a multicriteria optimization problem. Multicriteria based design enables us to do trade-off analysis for how much we must lose in one objective in order to do better in the other objective. For control design, the so called mixed criteria have already been adopted for practical implementation. As in control design, it would be meaningful to design adaptive observers with multiple useful criteria that can apply even to singular systems.

In this paper, we propose multicriteria adaptive observers for general singular systems with unknown time-varying parameters. For design of multicriteria adaptive observers, two criteria are employed to achieve robustness to disturbances and uncertainties. One is the H_∞ attenuation level which is an upper bound on the H_∞-norm of the transfer function from disturbances to estimation errors. The other is the upper bound of the ultimate region. These two criteria reflect how much disturbances and unknown parameters have effects on the estimation performance. Specially, the upper bound of the ultimate region makes the magnitudes of steady-state errors guaranteed to be upper bounded, which conflicts the H_∞ criterion and hence provides an optimal trade-off curve and achievable values.

The optimal trade-off curve between the ultimate bound and the H_∞ attenuation level is presented in the form of linear matrix inequalities (LMIs). Furthermore, the integrals of the error states are added for improving robustness to disturbances. If a singular matrix and time-varying parameters of the proposed multicriteria adaptive observers are set to be an identity matrix and constants, respectively, they reduce to existing adaptive observers for linear systems [13–15]. Simulation examples are presented to show the feasibility and the effectiveness of the proposed observers.

The paper is organized as follows: The description of multicriteria adaptive observers is given for a class of singular systems in Section 2. In Section 3, the design of multiobjective adaptive observers with integral effort is proposed. Finally, the simulation results are illustrated in Section 4 and the conclusion is drawn in Section 5.

2. Multicriteria Adaptive Observers

Let us consider the following singular system:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + B_u u(t) + B_\theta \phi(t,u,y)\theta(t) \\ &\quad + D_1 w(t), \\ y(t) &= Cx(t) + D_2 w(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $\theta(t) \in \mathbb{R}^p$ is the unknown time-varying parameter, $\phi(t,u,y)$ is the nonlinear term depending on the input and the output, $w(t)$ is the disturbance signal, $y(t)$ is the measured output signal, and E, A, B_u, B_θ, C, D_1, and D_2 are the system matrices of appropriate dimensions. For a well-defined singular system, the rank of E is assumed to be $r \le n$. The nonlinear term $\phi(t,u,y)$ is known and upper bounded as

$$\left|\phi(t,u,y)\right| \le \phi_{\max} \tag{2}$$

with a certain positive constant $\phi_{\max}$. In addition, it is assumed that uncertain parameters and their derivatives are upper bounded as

$$\begin{aligned} \|\theta\| &\le \alpha, \\ \|\dot{\theta}\| &\le \beta \end{aligned} \tag{3}$$

with positive constants α and β. Without loss of generality, the following conditions are also assumed to hold

$$\begin{aligned} \operatorname{rank}\begin{bmatrix} E \\ C \end{bmatrix} &= n, \\ \operatorname{rank}\begin{bmatrix} sE - A \\ C \end{bmatrix} &= n, \quad \forall s \in \mathbb{C}, \end{aligned} \tag{4}$$

where $\mathbb{C}$ is the set of complex numbers. Assumption (4) implies that the singular system (1) is observable. According to assumption (4), there exist nonsingular matrices T and N such that $TE + NC = I_n$, where $I_n \in \mathbb{R}^{n\times n}$ denotes an identity matrix. The general solution for T and N is given as

$$\begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^\dagger + J_1 \cdot \left(I - \begin{bmatrix} E \\ C \end{bmatrix} \begin{bmatrix} E \\ C \end{bmatrix}^\dagger \right), \tag{5}$$

where J_1 is an arbitrary matrix of appropriate dimension and the superscript † denotes pseudoinverse. To estimate both the state variables and the unknown parameters, the following functional observer can be constructed:

$$\begin{aligned} \dot{z}(t) &= Fz(t) + Gu(t) + Ky(t) + B_\theta \cdot \phi(t,u,y)\hat{\theta}(t), \\ \hat{x}(t) &= z(t) + Ny(t), \\ \hat{y} &= C\hat{x}(t), \end{aligned} \tag{6}$$

where $z(t)$ is the auxiliary variable of the observer, $\hat{\theta}$ is the estimated parameter value, and F, G, and K are constant matrices to be determined later on for guaranteeing observation. It follows then that we have the following error dynamics:

$$\begin{aligned} \dot{e}(t) &= (TA + FNC - KC)x(t) - F\hat{x}(t) \\ &\quad + (TB_u - G)u(t) + ND_2\dot{y}(t) \\ &\quad + TB_\theta\phi(t,u,y)\theta(t) - B_\theta\phi(t,u,y)\hat{\theta}(t) \\ &\quad + (FND_2 - KD_2 + TD_1)w(t), \end{aligned} \tag{7}$$

where $e(t) = x(t) - \hat{x}(t)$. If F, G, and K in (7) are chosen to satisfy the following conditions:

$$\begin{aligned} TA + FNC - KC &= F, \\ TB_u - G &= 0, \\ ND_2 &= 0, \end{aligned} \tag{8}$$

the error dynamics (7) becomes

$$\begin{aligned} \dot{e}(t) &= Fe(t) + M\phi\theta(t) + B_\theta\phi e_\theta(t) \\ &\quad + (FND_2 - KD_2 + TD_1)w(t), \end{aligned} \tag{9}$$

where $e_\theta(t) = \theta(t) - \hat{\theta}(t)$, $M = (T - I_n)B_\theta$, and the arguments of $\phi(t,u,y)$ are omitted for simplicity. Substituting (8) into (9) yields

$$\begin{aligned} \dot{e}(t) &= (TA + L_pC)e(t) + M\phi\theta(t) + B_\theta\phi e_\theta(t) \\ &\quad + D_3 w(t), \end{aligned} \tag{10}$$

where $L_p = FN - K$ and $D_3 = L_p D_2 + TD_1$. For the estimation of the unknown time-varying parameter $\theta(t)$, the following parameter update equation is constructed:

$$\dot{\widehat{\theta}}(t) = \rho_a \Gamma \phi^T UHCe(t) + \Gamma UHC\dot{e}(t) - \Gamma \rho_l \widehat{\theta}(t), \quad (11)$$

$$\begin{aligned} \dot{e}_\theta(t) &= \dot{\theta}(t) - \dot{\widehat{\theta}}(t) \\ &= \dot{\theta}(t) - \rho_a \Gamma \phi^T UHCe(t) - \Gamma UHC\dot{e}(t) \\ &\quad + \Gamma \rho_l \widehat{\theta}(t), \end{aligned} \quad (12)$$

where U, H are matrices to be designed, ρ_a is positive constant, Γ is a diagonal weight matrix for adaptation, and ρ_l is a leakage variable. The leakage term ρ_l is defined as [16]

$$\rho_l = \begin{cases} 0 & \text{if } \left\|\widehat{\theta}\right\| < \gamma_{\text{th}} \\ \rho_a \cdot \rho_1 & \text{if } \left\|\widehat{\theta}\right\| \geq \gamma_{\text{th}}, \end{cases} \quad (13)$$

where $\gamma_{\text{th}} \geq \alpha$ is a predefined threshold and ρ_1 is a positive constant. In the estimation of unknown time-varying parameters, the function of last term in (12) is to force the estimated variable $\widehat{\theta}$ to inside of the set $\|\widehat{\theta}\| < \gamma_{\text{th}}$. Therefore, the term is effective when the estimated $\widehat{\theta}$ exists outside of the set. If the estimated parameter value $\widehat{\theta}$ exists outside of the set, $\rho_a \rho_1$ determines how fast it converges. Though there is a possibility of small oscillations on the switching surface $\|\widehat{\theta}\| = \gamma_{\text{th}}$, the leakage term ensures an bounded parameter estimation error. Also, these oscillations do not occur under nominal conditions. Furthermore, the parameter estimation is assumed to be independent of disturbances. Then, $HD_2 = 0$ holds and the general solution is given as $H = J_2[I - D_2(D_2^T D_2)^{-1} D_2^T]$, where J_2 is an arbitrary matrix. To derive an observer gain considering the effect of disturbance, the H_∞ performance from the disturbance to the estimation error is defined as

$$\sup_{\|w\|_\infty \neq 0} \frac{\|Ze\|_\infty}{\|w\|_\infty}, \quad (14)$$

where sup denotes *supremum* and Z is a matrix with appropriate dimension. Now, we shall try to construct sufficient conditions for multiobjective observer based on quadratic Lyapunov functions.

Theorem 1. *For given positive scalars τ, ρ_a, η_1, η_2, η_3, and δ, if there exist matrices $P = P^T > 0$, $Q = Q^T > 0$, $R_1 = R_1^T > 0$, $R_2 = R_2^T > 0$, $R_3 = R_3^T > 0$, U, and X_1 and scalars k_1, k_2, and k_3 satisfying the following LMIs and the equality condition*

$$\min \quad \left[\tau \cdot \epsilon_{max} + (1-\tau) \cdot \gamma^2\right] \quad (15)$$

$$\begin{aligned} &\textit{subject to} \quad \begin{bmatrix} R_1 & PM\phi_{max} \\ \phi_{max}^T M^T P^T & k_1 \cdot I \end{bmatrix} > 0, \\ &\begin{bmatrix} R_2 & \Gamma^{-1} \\ \Gamma^{-T} & k_2 \cdot I \end{bmatrix} > 0, \end{aligned} \quad (16)$$

$$\begin{bmatrix} R_3 & B_\theta^T PM\phi_{max} \\ \phi_{max}^T M^T P^T B_\theta & k_3 \cdot I \end{bmatrix} > 0, \quad (17)$$

$$\Omega = \begin{bmatrix} \Omega_{11} & -\frac{1}{\rho_a} B_\theta (PTA + X_1 C)^T & PD_3 \\ * & -\frac{2}{\rho_a} B_\theta^T PB_\theta \phi_{max} + \frac{1}{\rho_a \eta_2} R_2 + \frac{1}{\rho_a \eta_3} R_3 & -\frac{1}{\rho_a} B_\theta^T PD_3 \\ * & * & -\gamma^2 I \end{bmatrix} < -\delta \cdot I, \quad (18)$$

$$B_\theta^T P = UHC, \quad (19)$$

where

$$\begin{aligned} \epsilon_{max} &= \eta_1 \alpha^2 k_1 + \frac{\eta_2}{\rho_a} \beta^2 k_2 + \frac{\eta_3}{\rho_a} \alpha^2 k_3, \\ \Omega_{11} &= A^T T^T P + PTA + C^T X_1^T + XC + \frac{1}{\eta_1} R_1 + Z^T Z, \end{aligned} \quad (20)$$

and $*$ denotes the entry of a symmetric matrix, then, the state estimation error and the parameter estimation error are uniformly ultimately bounded for an ultimate ellipsoidal set ϵ_{max} with H_∞ attenuation level γ. Moreover, the observer gain is chosen to be $L_p = P^{-1} X_1$.

Proof. Choose the following Lyapunov function of a quadratic form:

$$V(e(t), e_\theta(t)) = e^T(t) Pe(t) + \frac{1}{\rho_a} e_\theta^T(t) \Gamma^{-1} e_\theta(t). \quad (21)$$

Differentiating the Lyapunov function (21) along the state trajectory yields

$$\begin{aligned}\dot{V}(t) &= 2e^TPFe + 2e^TPM\phi\theta + 2e^TPB_\theta\phi e_\theta \\ &\quad + 2e^TPD_3w + \frac{2}{\rho_a}e_\theta^T\Gamma^{-1}\dot{\theta} - 2e_\theta^T\phi^TUHCe \\ &\quad + \frac{2}{\rho_a}e_\theta^T\rho_l\widehat{\theta} \\ &\quad - \frac{2}{\rho_a}e_\theta^TUHC\left(Fe + M\phi\theta + B_\theta\phi e_\theta + D_3w\right) \\ &= 2e^TPFe + 2e^TPM\phi\theta + 2e^TPD_3w + \frac{2}{\rho_a}e_\theta^T\Gamma^{-1}\dot{\theta} \\ &\quad + \frac{2}{\rho_a}e_\theta^T\rho_l\widehat{\theta} \\ &\quad - \frac{2}{\rho_a}e_\theta^TB_\theta^TP\left(Fe + M\phi\theta + B_\theta\phi e_\theta + D_3w\right),\end{aligned} \tag{22}$$

if $B_\theta^TP = UHC$ is satisfied. The terms in (22) have upper bounds as follows:

$$\begin{aligned}\frac{2}{\rho_a}e_\theta^T\rho_l\widehat{\theta} &\le \frac{2\rho_l}{\rho_a}\left(\|\theta\|\left\|\widehat{\theta}\right\| - \left\|\widehat{\theta}\right\|\left\|\widehat{\theta}\right\|\right) \\ &= \frac{2\rho_l}{\rho_a}\left(\gamma_{\text{th}} - \left\|\widehat{\theta}\right\|\right)\left\|\widehat{\theta}\right\| \le 0, \\ 2e^TPM\phi\theta &\le \frac{1}{\eta_1}e^TR_1e \\ &\quad + \eta_1\theta^T\phi^TM^TP^TR_1^{-1}PM\phi\theta, \\ \frac{2}{\rho_a}e_\theta^T\Gamma^{-1}\dot{\theta} &\le \frac{1}{\rho_a\eta_2}e_\theta^TR_2e_\theta + \frac{\eta_2}{\rho_a}\dot{\theta}^T\Gamma^{-T}R_2^{-1}\Gamma^{-1}\dot{\theta}, \\ -\frac{2}{\rho_a}e_\theta^TB_\theta^TPM\phi\theta &\le \frac{1}{\rho_a\eta_3}e_\theta^TR_3e_\theta \\ &\quad + \frac{\eta_3}{\rho_a}\theta^T\phi^TM^TP^TB_\theta R_3^{-1}B_\theta^TPM\phi\theta\end{aligned} \tag{23}$$

which comes from the following well-known inequality:

$$2x^Ty \le \frac{1}{\eta}x^TSx + \eta y^TS^{-1}y, \quad S > 0. \tag{24}$$

Putting together inequalities in (23) and ignoring the effect of disturbances (i.e., $w = 0$), we have

$$\begin{aligned}\dot{V}(t) &\le 2e^TP\left(TA + L_pC\right)e \\ &\quad - \frac{2}{\rho_a}e_\theta^TB_\theta^TP\left(TA + L_pC\right)e \\ &\quad - \frac{2}{\rho_a}e_\theta^TB_\theta^TPB_\theta\phi_{\max}e_\theta + \frac{1}{\eta_1}e^T(t)R_1e(t) \\ &\quad + \frac{1}{\rho_a\eta_2}e_\theta^T(t)R_2e_\theta(t) + \frac{1}{\rho_a\eta_3}e_\theta^TR_3e_\theta + \epsilon,\end{aligned} \tag{25}$$

where ϵ is defined by

$$\begin{aligned}\epsilon = \max\Big[&\eta_1\alpha^2\lambda_{\max}\left(\phi_{\max}^TM^TP^TR_1^{-1}PM\phi_{\max}\right) \\ &+ \frac{\eta_2}{\rho_a}\beta^2\lambda_{\max}\left(\Gamma^{-T}R_2^{-1}\Gamma^{-1}\right) \\ &+ \frac{\eta_3}{\rho_a}\alpha^2\lambda_{\max}\left(\phi_{\max}^TM^TP^TB_\theta R_3^{-1}B_\theta^TPM\phi_{\max}\right)\Big].\end{aligned} \tag{26}$$

The right hand side of inequality (25), except for ϵ, can be converted into an LMI and upper bounded as follows:

$$\begin{aligned}\Xi = \begin{bmatrix}\Xi_1 & -\dfrac{1}{\rho_a}B_\theta\left(PTA + X_1C\right)^T \\ * & -\dfrac{2}{\rho_a}B_\theta^TPB_\theta\phi_{\max} + \dfrac{1}{\rho_a\eta_2}R_2 + \dfrac{1}{\rho_a\eta_3}R_3\end{bmatrix} \\ < -\delta \cdot I,\end{aligned} \tag{27}$$

where the Schur complement is used, $PL_p = X_1$, $\Xi_1 = A^TT^TP + PTA + C^TX_1^T + XC + (1/\eta_1)R_1$, and δ is a design parameter to be chosen to be a small positive constant. If $\Xi < -\delta \cdot I$ is satisfied, then, inequality (25) can be expressed as

$$\dot{V}(e(t)) < -\delta \cdot \left(\|e\| + \|e_\theta\|\right) + \epsilon. \tag{28}$$

It implies that $\dot{V}(e(t), e_\theta(t)) < 0$ for $\delta \cdot (\|e(t)\| + \|e_\theta(t)\|) > \epsilon$. When the estimation error exists outside of the bound, it approaches the inside of the bound and then it stays there according to the Lyapunov stability theory. Therefore, $e(t)$, $e_\theta(t)$ converge to the inside of a set parameterized by ϵ; that is, $\{(e, e_\theta) \mid \|e\| + \|e_\theta\| < \epsilon/\delta\}$. It means the error dynamics is uniformly ultimately bounded with the ultimate bound ϵ/δ.

Now, the existence of disturbances is taken into account (the case of $w \ne 0$). For the H_∞ performance $\sup_{\|w\|_2 \ne 0}(\|Ze\|_2/\|w\|_2) \le \gamma$, the following inequality is considered with the derivative of Lyapunov function in (22).

$$\dot{V}(t) + e^T(t)Z^T(t)Z(t)e(t) - \gamma^2w^T(t)w(t) < 0. \tag{29}$$

Using Schur complement, (29) is equivalent to (18). From (28), the ultimate bound region ϵ is given as $\max[\eta_1\alpha^2\lambda_{\max}(\phi_{\max}^TM^TP^TR_1^{-1}PM\phi_{\max}) + (\eta_2/\rho_a)\beta^2\lambda_{\max}(\Gamma^{-1}R_2^{-1}\Gamma^{-1}) + (\eta_3/\rho_a)\alpha^2\lambda_{\max}(\phi_{\max}^TM^TP^TB_\theta R_3^{-1}B_\theta^TPM\phi_{\max})]$. Putting k_1, k_2, and k_3 as an upper bound of each term yields

$$\begin{aligned}\phi_{\max}^TM^TP^TR_1^{-1}PM\phi_{\max} &\le k_1 \cdot I, \\ \Gamma^{-T}R_2^{-1}\Gamma^{-1} &\le k_2 \cdot I, \\ \phi_{\max}^TM^TP^TB_\theta R_3^{-1}B_\theta^TPM\phi_{\max} &\le k_3 \cdot I.\end{aligned} \tag{30}$$

Applying the Schur complement, inequalities (30) are transformed to (16) to (17) in Theorem 1. Then, the maximum ultimate bound is given as $\epsilon_{\max} = \eta_1\alpha^2k_1 + (\eta_2/\rho_a)\beta^2k_2 + (\eta_3/\rho_a)\alpha^2k_3$. Considering the maximum bound $\epsilon_{\max}$ and H_∞ performance γ, multiobjective function can be constructed as

$$\tau \cdot \epsilon_{\max} + (1 - \tau) \cdot \gamma^2, \tag{31}$$

where $0 \le \tau \le 1$ is a weight parameter. This completes the proof. □

3. Multicriteria Adaptive Observers with Integral Effort

In this section, the multiobjective adaptive observer involving integral action is presented to improve steady-state accuracy and attain the robustness to exogenous disturbances, which is of the following form:

$$\begin{aligned}\dot{z}(t) &= Fz(t) + Gu(t) + Ky(t) + L_I\zeta + B_\theta \\ &\quad \cdot \phi(t,u,y)\hat{\theta}(t), \\ \dot{\zeta}(t) &= -A_\zeta\zeta + (y(t) - \hat{y}(t)), \\ \hat{x}(t) &= z(t) + Ny(t), \\ \hat{y} &= C\hat{x}(t),\end{aligned} \tag{32}$$

where ζ is the integral of the estimation error. The proposed multicriteria adaptive observer (32) with integral effort yields the following error dynamics:

$$\begin{aligned}\dot{e} &= Fe - L_I\zeta + M\phi\theta + B_\theta\phi e_\theta + (L_pD_2 + TD_1)w, \\ \dot{\zeta} &= -A_\zeta\zeta + Ce + D_2w.\end{aligned} \tag{33}$$

The following theorem tells us that the multicriteria adaptive observer (32) is guaranteed to achieve the H_∞ attenuation level and the upper bound of the ultimate invariant set if some LMI conditions are met. $\overline{\epsilon}_{\max}, \overline{\gamma}$ is used for the design of multicriteria with integral effort in order to distinguish them from $\epsilon_{\max}, \gamma$ for the one without integral term.

Theorem 2. *For given scalars $0 \le \tau \le 1$, $\eta_1 > 0$, $\eta_2 > 0$, $\eta_3 > 0$, $\rho_a > 0$, and $\delta > 0$, if there exist matrices $\overline{P} = \overline{P}^T > 0$, $\overline{Q} = \overline{Q}^T > 0$, $\overline{R}_1 = \overline{R}_1^T$, $\overline{R}_2 = \overline{R}_2^T$, $\overline{R}_3 = \overline{R}_3^T$, $\overline{U}$, $\overline{X}_1$, $\overline{X}_2$, and $\overline{Y}$ and scalars $\overline{k}_1$, $\overline{k}_2$, and $\overline{k}_3$ such that*

$$\min \quad \tau\overline{\epsilon}_{max} + (1-\tau)\overline{\gamma}^2 \tag{34}$$

$$\text{subject to} \quad \begin{bmatrix} \overline{R}_1 & \overline{P}M\phi_{max} \\ \phi_{max}^T M^T \overline{P}^T & \overline{k}_1 \cdot I \end{bmatrix} > 0, \qquad \begin{bmatrix} \overline{R}_2 & \Gamma^{-1} \\ \Gamma^{-T} & \overline{k}_2 \cdot I \end{bmatrix} > 0, \tag{35}$$

$$\begin{bmatrix} \overline{R}_3 & B_\theta^T \overline{P}M\phi_{max} \\ \phi_{max}^T M^T \overline{P}^T B_\theta & \overline{k}_3 \cdot I \end{bmatrix} > 0, \tag{36}$$

$$\begin{bmatrix} \Upsilon_1 & C^T\overline{Q} - \overline{X}_2 & -\frac{1}{\rho_a}(\overline{P}TA + \overline{X}_1C)^T B_\theta + B_\theta^T\overline{P} & \overline{P}TD_1 + \overline{X}_1D_2 \\ * & -2\overline{Y} & -\frac{1}{\rho_a}\overline{X}_2B_\theta & \overline{Q}D_2 \\ * & * & -\frac{2}{\rho_a}B_\theta^T\overline{Q}B_\theta\phi_{max} + \frac{1}{\rho_a\eta_2}\overline{R}_2 + \frac{1}{\rho_a\eta_3}\overline{R}_3 & -\frac{1}{\rho_a}B_\theta^T\overline{P}D_3 \\ * & * & * & -\overline{\gamma}^2 I \end{bmatrix} < -\delta I, \tag{37}$$

$$B_\theta^T\overline{P} = \overline{U}HC, \tag{38}$$

where

$$\begin{aligned}\overline{\epsilon}_{max} &= \eta_1\alpha^2\overline{k}_1 + \frac{\eta_2}{\rho_a}\beta^2\overline{k}_2 + \frac{\eta_3}{\rho_a}\alpha^2\overline{k}_3, \\ \Upsilon_1 &= A^TT^T\overline{P} + \overline{P}TA + C^T\overline{X}_1^T + \overline{X}_1C + \frac{1}{\eta_1}\overline{R}_1 \\ &\quad + Z^TZ,\end{aligned} \tag{39}$$

then, the error dynamic (33) is ultimately bounded with an upper bound $\overline{\epsilon}_{max}$ and satisfies the H_∞ performance with the $\overline{\gamma}$ attenuation level. The observer gain is computed as $L_p = \overline{P}^{-1}\overline{X}_1$, $L_I = \overline{P}^{-1}\overline{X}_2$, and $A_\zeta = \overline{Q}^{-1}\overline{Y}$.

Proof. Choose a Lyapunov candidate function as follows:

$$\begin{aligned}V(t) &= e^T(t)\overline{P}e(t) + \zeta^T(t)\overline{Q}\zeta(t) \\ &\quad + \frac{1}{\rho_a}e_\theta^T(t)\Gamma^{-1}e_\theta(t).\end{aligned} \tag{40}$$

Differentiating the Lyapunov function along the state trajectory with the condition $B_\theta^T \overline{P} = \overline{U}HC$ results in

$$
\begin{aligned}
\dot{V} &= 2e^T\overline{P}Fe - 2e^T\overline{P}L_I\zeta + 2e^T\overline{P}M\phi\theta + 2e^T\overline{P}D_3 w \\
&\quad - 2\zeta^T\overline{Q}A_\zeta\zeta + 2\zeta^T\overline{Q}Ce + 2\zeta^T\overline{Q}D_2 w + \frac{2}{\rho_a}e_\theta^T\Gamma^{-1}\dot{\theta} \\
&\quad + \frac{2}{\rho_a}e_\theta^T\rho_l\widehat{\theta} \\
&\quad - \frac{2}{\rho_a}e_\theta^T B_\theta^T\overline{P}\left(Fe - L_I\zeta + M\phi\theta + B_\theta\phi e_\theta + D_3 w\right),
\end{aligned}
\tag{41}
$$

where equality $B_\theta^T\overline{P} = \overline{U}HC$ is used. For now, the case of $w = 0$ is considered.

Using (24) and Schur complement,

$$
\dot{V} = \Upsilon + \epsilon, \tag{42}
$$

where

$$
\begin{aligned}
\Upsilon &= \begin{bmatrix} \Upsilon_1 & C^T\overline{Q} - \overline{X}_2 & -\frac{1}{\rho_a}\left(\overline{P}TA + \overline{X}_1 C\right)^T B_\theta + B_\theta^T\overline{P} \\ * & -2\overline{Y} & -\frac{1}{\rho_a}\overline{X}_2 B_\theta \\ * & * & -\frac{2}{\rho_a}B_\theta^T\overline{Q}B_\theta\phi_{\max} + \frac{1}{\rho_a\eta_2}\overline{R}_2 + \frac{1}{\rho_a\eta_3}\overline{R}_3 \end{bmatrix}, \\
\Upsilon_1 &= A^T T^T\overline{P} + \overline{P}TA + C^T\overline{X}_1^T + \overline{X}_1 C + \frac{1}{\eta_1}\overline{R}_1, \\
\overline{X}_1 &= \overline{P}L_p, \\
\overline{X}_2 &= \overline{P}L_I, \\
\overline{Y} &= \overline{Q}A_\zeta, \\
\epsilon &= \max\left[\eta_1\alpha^2\lambda_{\max}\left(M^T\overline{P}^T\overline{R}_1^{-1}\overline{P}M\right)\right. \\
&\quad + \frac{\eta_2}{\rho_a}\beta^2\lambda_{\max}\left(\Gamma^{-T}\overline{R}_2^{-1}\Gamma^{-1}\right) \\
&\quad \left. + \frac{\eta_3}{\rho_a}\alpha^2\lambda_{\max}\left(M^T\overline{P}^T B_\theta\overline{R}_3^{-1}B_\theta^T\overline{P}M\right)\right].
\end{aligned}
\tag{43}
$$

Then, the right hand side of inequality (42), except for ϵ, can be converted into (37) using the Schur complement and upper bounded. If $\Upsilon < -\delta \cdot I$ is satisfied, then, the inequality can be expressed as

$$
\dot{V}(e(t)) < -\delta \cdot \left(\|e(t)\|_2 + \|\zeta(t)\|_2 + \|e_\theta(t)\|_2\right) + \epsilon. \tag{44}
$$

Therefore, $e(t)$, $\zeta(t)$, and $e_\theta(t)$ are uniformly ultimately bounded with the ultimate bound ϵ. The rest part is similar to that of Theorem 1, so it is omitted for brevity. This completes the proof. □

Remark 3. Since singular systems has a complicated structure, they provide more challenging issues. The proposed adaptive observer is more generalized than existing ones [7, 8, 13] that can be only applied to linear systems. Choosing $T = I$, $N = 0$, it can be applied to standard linear systems. Further, the proposed one deals with time-varying parameters.

Remark 4. The proposed observer offers flexibility between two criteria, an ultimate bound and H_∞ performance, using a multiobjective approach. Until now, such design approaches for adaptive observers in singular systems with unknown time-varying parameters have not been studied at all.

4. Numerical Simulation

In this section, two examples for numerical simulations are considered to verify the effectiveness of the proposed multicriteria adaptive observers.

4.1. Example 1: A Second-Order Singular System. At the first example, the following second-order singular system is considered:

$$
\begin{aligned}
E &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\
A &= \begin{bmatrix} -5 & 1 \\ -0.5 & -1 \end{bmatrix}, \\
B_u &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
B_\theta &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
C &= \begin{bmatrix} 1 & -1 \end{bmatrix}, \\
D_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
D_2 &= 0.
\end{aligned}
\tag{45}
$$

The time-varying parameter is chosen to be $\theta = 0.3\sin(t)$ and ϕ is taken to be the sinusoidal function $\sin(5t)$. The parameters are chosen as $\rho_a = 10$, $\eta_1 = \eta_2 = \eta_3 = 1$, and $\delta = 0.01$. Applying Theorem 2, the optimal gains of the multiobjective proportional-integral adaptive observer with $\tau = 0.1$ are computed to be

$$
\begin{aligned}
L_P &= \begin{bmatrix} -10.2759 \\ -5.8751 \end{bmatrix}, \\
L_I &= \begin{bmatrix} 1.5087 \\ 0.4954 \end{bmatrix}, \\
Az &= 0.6118.
\end{aligned}
\tag{46}
$$

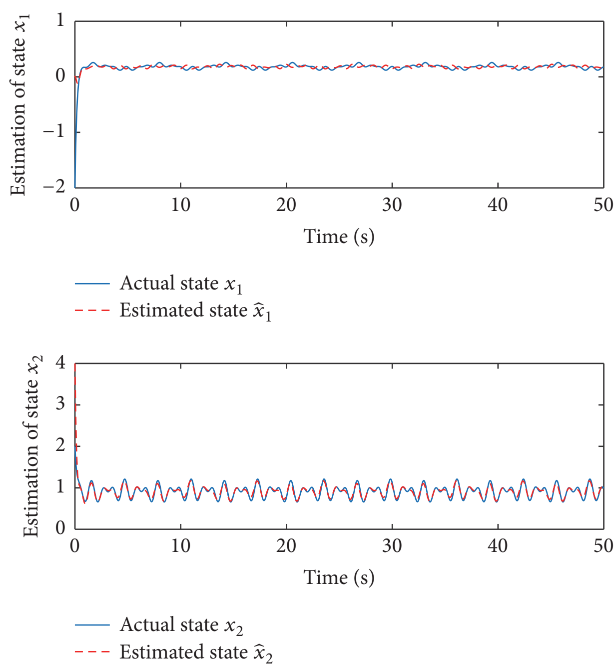

FIGURE 1: The state trajectories.

The solutions are provided with matrices

$$\begin{aligned} \overline{P} &= \begin{bmatrix} 0.2059 & -0.1887 \\ -0.1887 & 0.1801 \end{bmatrix}, \\ \overline{Q} &= 0.3256, \\ \overline{U} &= -0.1886, \\ \overline{Y} &= 0.1992, \\ \overline{X}_1 &= \begin{bmatrix} -1.0077 \\ 0.8811 \end{bmatrix}, \\ \overline{X}_2 &= \begin{bmatrix} 0.2172 \\ -0.1955 \end{bmatrix}. \end{aligned} \tag{47}$$

In the presence of external disturbance $w(t) = 0.2\sin(10t)$, the observer state tracks along a real state. By solving the multiobjective optimization problem, the optimal H_∞ performance index is given as $\sup_{\|w\|_2 \neq 0}(\|\bar{e}\|_2/\|w\|_2) \leq 0.01839$ and the optimal upper bound $\bar{e}_{\max} = 0.1855$ is provided. Then, the system response curves of the system with the initial values $x(0) = [-2, 2]$ are shown in Figure 1, which include the trajectories of state and estimated states. The oscillations in the estimation of states are caused by the external disturbances due to $0.2\sin(10t)$ and nonlinearity $\phi(t, x, u)$. The parameter estimation curve is illustrated in Figure 2.

4.2. Example 2: Leontief Model. In economics, the Leontief model describes the total production of the output required from each different industry to meet all demands. The model

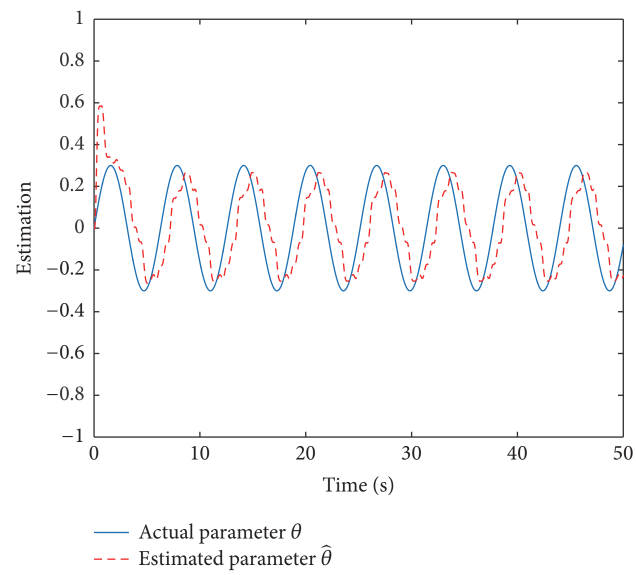

FIGURE 2: The time-varying parameter and its estimate.

has been widely considered to predict the proper level of production of several types of goods. The state x represents the production of each industry, the matrix A corresponds to the rate of production, E is the stock placement of commodities, the input $B_u u(t)$ presents the known supply rate, θ is the external supply, the disturbance $w(t)$ represents the uncertain industrial supply, and y corresponds to the production of commodities available for evaluation. For simulations, the system matrices are considered as follows:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A = \frac{1}{20}\begin{bmatrix} -6 & 2 & 1 & 1 & 2 \\ 4 & -4 & 2 & 3 & 2 \\ 1 & 1 & -5 & 2 & 0 \\ 1 & 2 & 1 & -5 & 1 \\ 1 & 1 & 2 & 1 & -5 \end{bmatrix},$$

$$B_u = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

$$B_\theta = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}, \tag{48}$$

with $e(0) = [10, 20, 20, 17.5, 17.5]$, $\phi(t, u, y) = \sin(2t)$, $\theta(t) = \sin(t)$, and $w(t) = 0.1\cos(10t)$. The resulting LMI solutions given by Theorem 2 with $\tau = 0.4$ are

$$\overline{P} = \begin{bmatrix} 0.6555 & -0.2609 & 0.1282 & 0.0892 & -0.1320 \\ -0.2609 & 0.2901 & 0.2008 & -0.1505 & 0.0713 \\ 0.1282 & 0.2008 & 0.4603 & -0.2110 & -0.0026 \\ 0.0892 & -0.1505 & -0.2110 & 0.2931 & 0.2255 \\ -0.1320 & 0.0713 & -0.0026 & 0.2255 & 0.5065 \end{bmatrix},$$

$$\overline{Q} = \begin{bmatrix} 0.4192 \end{bmatrix},$$

$$\overline{U} = \begin{bmatrix} 0.0574 \end{bmatrix},$$

$$\overline{Y} = \begin{bmatrix} 0.2760 \end{bmatrix},$$

$$A_\zeta = \begin{bmatrix} 0.6583 \end{bmatrix},$$

$$\overline{X}_1 = \begin{bmatrix} -0.9631 \\ -0.0849 \\ -1.0807 \\ -0.0330 \\ -0.1596 \end{bmatrix},$$

$$\overline{X}_2 = \begin{bmatrix} 0.2269 \\ -0.0004 \\ 0.2238 \\ -0.0032 \\ -0.0130 \end{bmatrix},$$

$$L_p = \begin{bmatrix} 5.5932 \\ 5.3555 \\ -11.5155 \\ -11.5722 \\ 5.4816 \end{bmatrix},$$

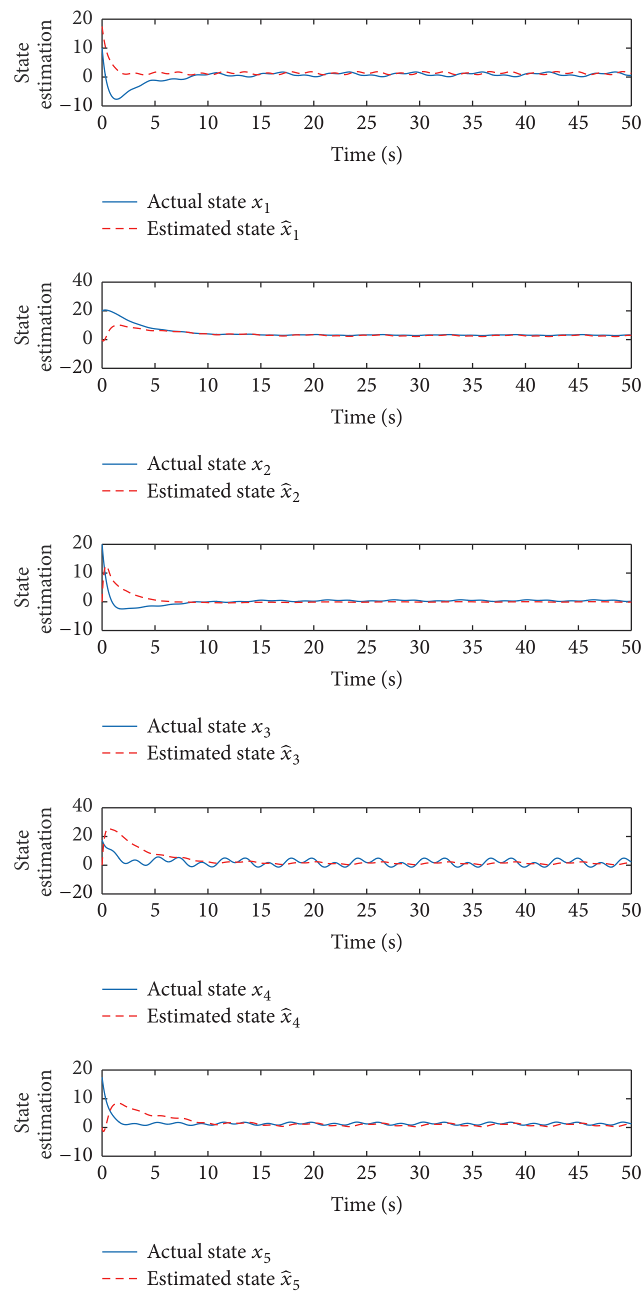

Figure 3: The estimation errors.

$$L_I = \begin{bmatrix} -1.3456 \\ -1.3087 \\ 2.6688 \\ 2.7154 \\ -1.3873 \end{bmatrix}. \tag{49}$$

The optimal value of multiobjective function is computed to be 0.8346. $\overline{\gamma}^2 = 0.4773$ and $\overline{\epsilon}_{\max} = 1.3706$ are computed. $Z = C$ is chosen for the H_∞ performance index. In the presence of the disturbances, the estimation errors converge to zero. The convergence of error dynamics is presented in Figure 3.

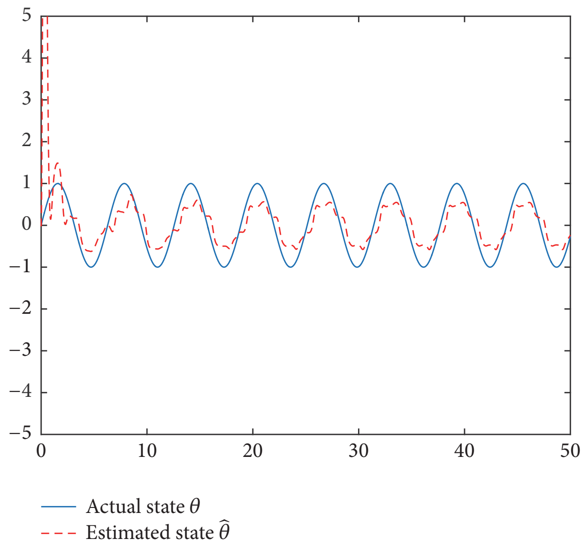

Figure 4: The parameter estimation.

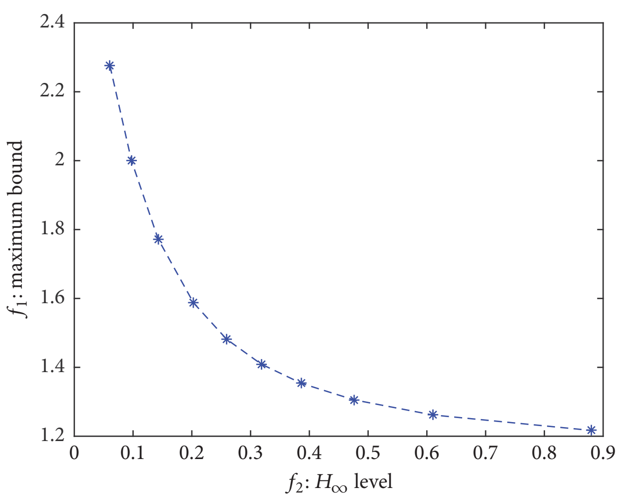

Figure 5: The optimal trade-off curve for the multicriteria adaptive observer.

Figure 4 shows the trajectory of the parameter estimation errors. To show the trade-off between the ultimate bound $\bar{\epsilon}_{max}$ and the H_∞ attenuation level $\bar{\gamma}$, the optimal solutions are solved for various τ values. Plotting these optimal solutions, we obtain the Pareto optimal points as described in Figure 5. As shown in Figure 5, $\bar{\epsilon}_{max}$ seems to be inversely proportional to $\bar{\gamma}^2$. Figure 6 displays the comparison of the transient trajectories for different adaptive gains. To follow real parameter θ as fast as possible, a high adaptive gain is needed. However, if the adaptive gain is too large, it causes oscillations in the transient period but it has a good tracking performance for parameters as the case of $\rho_a = 20$. Conversely, if the adaptive gain is too small, there is comparably small oscillations in the transient period but the observer provides a poor tracking performance for parameters as the case of $\rho_a = 5$. Therefore, the adaptive gain should be appropriately chosen.

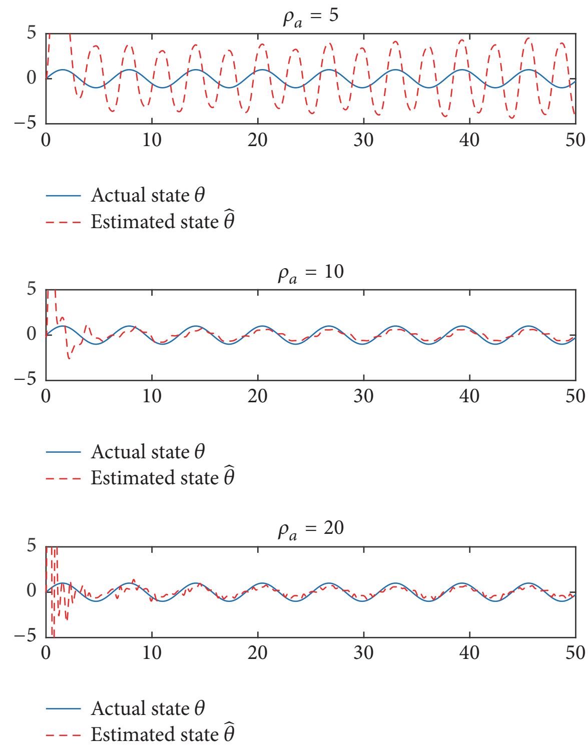

Figure 6: The comparison of parameter estimation with different adaptive gains.

5. Conclusion

Multicriteria adaptive observers were designed according to two criteria for the H_∞ attenuation level of disturbances and the upper bound of the ultimate invariant set. The corresponding cost functions are scalarized into a single one and then the Pareto optimal solutions are obtained with Lyapunov stability in order to provide a good compromising solution. It was shown through numerical simulations that the proposed multicriteria adaptive observers have the good tracking ability.

For adaptive observers for general singular systems, other criteria can be easily taken into consideration by extending the proposed design scheme. It is believed that the proposed observers could be applied to fault detection, unknown input estimation, disturbance estimation, and so on.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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