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A New Control Chart for Monitoring Reliability Using Sudden Death Testing Under Weibull Distribution

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ABSTRACT In this paper, a new control chart using sudden death testing is designed by assuming that the lifetime/failure time of the product follows the Weibull distribution. The structure of the proposed chart is presented. The control chart coefficient is determined using some specified average run length for the in control process and the shifted process. Simulation study is given for the illustration purpose.

INDEX TERMS Life test, Weibull distribution, average run length.

I. INTRODUCTION

Statistical process control (SPC) has been considered a powerful tool in quality management decisions. Control chart and acceptance sampling plans are considered as integral parts of SPC. The earlier is used to monitor the manufacturing process and later one is used for the inspection of the finished product. The manufacturing process is monitoring through two control limits called lower control limit (LCL) and upper control limit (UCL). Shewhart control chart is considered to be more effective in detecting large shift in the process. A process beyond these control limits is said to be out-of-control. During the manufacturing, the process can shift from target value due to several factors such as temperature, workers and machines etc. [1] A timely indication is necessary to indicate the shift in the process to avoid the defective/non-conforming items. The control charts help in taking corrective decision for improving the quality of the product. The control charts have much applications in variety of fields including health care, nuclear engineering and epidemiology etc. see, for example [2]–[4].

The control chart can be classified into two classes such as attribute control chart and variable control chart. If the plotting statistic is computed from the attribute data, then attribute control chart is used to monitor the process. The variable control charts are used when the data is obtained from the measurement process. Both control charts have much application in the industry. Attribute chart is easy to apply but less informative than the variable control charts. Several authors designed and discussed the applications of control charts in verity of fields including for example, see [5]–[16] and [17]. Due to high reliability of product, it may not possible to wait failure of all products being tested for inspection. Recently, [18] designed attribute control chart for Weibull distribution for truncated life test. More details about the control chart using the Weibull distribution can be seen in [17], [19]–[24] and [25]. The main objective of this paper to develop a complete structure of proposed chart using the sudden death testing. We will present the necessary measures to evaluate the performance of proposed chart. A simulation study will be given for illustration purpose.

II. DESIGN OF PROPOSED CONTROL CHART

Suppose that the lifetime of a part (denoted by X) follows a Weibull distribution with shape parameter m and scale parameter λ such that the cumulative distribution function is given by

$$F(x) = 1 - \exp(-(\lambda x)^{m}) \tag{1}$$

According to [26], sudden death testing is frequently adopted by parts manufacturers to reduce testing time. In this testing,a sample of items are distributed into g groups having r items in one group. By exploring the literature, we note that there is no work on the designing ofa control chart using the sudden death testing. The operational procedure of the proposed control chart is stated as follows

Step-1: Select a random sample of size n from a lot and distribute r items into g groups so that n = rg.

TABLE 1. The values of ARL when $ARL_0 = 300$; $\lambda = 0.5$.

	r=5				r=10			
k	3.102291	3.102291	3.102289	3.10229	3.489702	3.4897	3.489701	3.489701
g	3	3	3	3	4	4	4	4
m	1	2	2.5	3	1	2	2.5	3
c	ARL							
1	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.99	291.11	282.48	278.25	274.10	288.20	276.86	271.36	265.97
0.98	282.39	265.81	257.89	250.21	276.75	255.30	245.22	235.52
0.97	257.29	220.66	204.36	189.26	244.45	199.19	179.82	162.34
0.95	241.41	194.28	174.29	156.36	224.54	168.10	145.45	125.87
0.93	218.84	159.67	136.40	116.53	197.00	129.43	104.93	85.09
0.9	184.43	113.46	89.02	69.87	156.84	82.11	59.47	43.10
0.85	153.85	79.01	56.68	40.70	123.18	50.75	32.66	21.09
0.8	126.85	53.81	35.12	22.99	95.27	30.49	17.37	10.00
0.75	103.23	35.74	21.14	12.59	72.41	17.77	8.97	4.67
0.7	65.19	14.48	7.01	3.55	39.32	5.55	2.36	1.28
0.6	37.94	5.20	2.20	1.22	19.22	1.74	1.03	1.00
0.5	19.67	1.79	1.04	1.00	8.18	1.01	1.00	1.00
0.3	8.60	1.01	1.00	1.00	2.96	1.00	1.00	1.00
0.2	2.93	1.00	1.00	1.00	1.14	1.00	1.00	1.00
0.1	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 2. The values of ARL when $ARL_0 = 370$; $\lambda = 0.5$.

	r=5				r=10			
k	3.146532	3.146532	3.146532	3.53055	3.530574	3.530612	3.530554	3.146537
g	3	3	3	4	4	4	4	3
m	1	2	2.5	3	1	2	2.5	3
c	ARL							
1	370.00	370.00	370.00	370.00	370.05	370.12	370.01	370.01
0.99	359.03	348.38	343.17	328.02	355.49	341.57	334.68	338.05
0.98	348.27	327.82	318.05	290.46	341.36	314.96	302.42	308.58
0.97	317.30	272.12	252.00	200.17	301.50	245.71	221.74	233.38
0.95	297.71	239.56	214.91	155.17	276.94	207.33	179.34	192.80
0.93	269.87	196.87	168.16	104.86	242.96	159.61	129.34	143.66
0.9	227.42	139.86	109.71	53.06	193.41	101.22	73.25	86.09
0.85	189.68	97.36	69.81	25.90	151.87	62.51	40.18	50.10
0.8	156.38	66.26	43.22	12.21	117.43	37.51	21.31	28.25
0.75	127.24	43.97	25.96	5.63	89.23	21.81	10.95	15.42
0.7	80.31	17.74	8.52	1.41	48.40	6.72	2.77	4.25
0.6	46.69	6.29	2.58	1.00	23.60	2.00	1.07	1.33
0.5	24.15	2.07	1.08	1.00	9.97	1.02	1.00	1.00
0.3	10.48	1.03	1.00	1.00	3.52	1.00	1.00	1.00
0.2	3.48	1.00	1.00	1.00	1.23	1.00	1.00	1.00
0.1	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Step-2: Perform sudden death testing and observe Y_i, the time to the first failure from the ith group (i = 1, 2, ... g). Calculate the quantity $v = \sum_{i=1}^{g} Y_i^m$ and transform it to $v^* = v^{1/3}$.

Step-3: Plot v^* on the control chart. Declare the process in control if $LCL \le v^* \le UCL$. Declare the process out-of-control if $v^* > UCL$ or $v^* < LCL$.

The proposed control chart is based on two control limits, namely LCL and UCL. As in Jun et al. (2006), the quantity v in Step 2 follows a gamma distribution with shape parameter g and scale parameter $r\lambda^m$. In order to set up symmetric Shewhart-type control limits, we need to transform v to a random variable having a symmetric distribution. Wilson and Hilferty [29] suggested that a gamma random variable can be transformed to an approximate normal variable through power transformation. In fact, the transformation of $v^* = v^{1/3}$ leads to an approximate normal distribution with mean

$$\mu_{v^*} = \frac{(r\lambda^m)^{1/3}\,\Gamma(g + 1/3)}{\Gamma(g)} \quad (2)$$

TABLE 3. The values of ARL when $ARL_0 = 300$; $\lambda = 1$.

	r=5				r=10			
k	3.4897	3.102294	3.102294	3.489702	3.102294	3.489701	3.489705	3.489783
g	4	3	3	4	3	4	4	4
m	1	2	2.5	3	1	2	2.5	3
c	ARL							
1	300.00	300.01	300.01	300.00	300.01	300.00	300.01	300.13
0.99	288.20	282.48	278.26	265.98	291.11	276.86	271.37	266.08
0.98	276.75	265.82	257.90	235.53	282.39	255.30	245.22	235.62
0.97	244.45	220.67	204.36	162.34	257.29	199.20	179.83	162.41
0.95	224.54	194.28	174.29	125.87	241.41	168.10	145.46	125.92
0.93	197.00	159.67	136.40	85.09	218.84	129.43	104.93	85.12
0.9	156.84	113.46	89.02	43.10	184.44	82.11	59.47	43.12
0.85	123.18	79.02	56.68	21.09	153.85	50.75	32.66	21.09
0.8	95.26	53.81	35.13	10.00	126.86	30.49	17.37	10.00
0.75	72.41	35.74	21.14	4.67	103.23	17.77	8.97	4.67
0.7	39.32	14.48	7.01	1.28	65.19	5.55	2.36	1.28
0.6	19.22	5.20	2.20	1.00	37.94	1.74	1.03	1.00
0.5	8.18	1.79	1.04	1.00	19.67	1.01	1.00	1.00
0.3	2.96	1.01	1.00	1.00	8.60	1.00	1.00	1.00
0.2	1.14	1.00	1.00	1.00	2.93	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.04	1.00	1.00	1.00

TABLE 4. The values of ARL when $ARL_0 = 370$; $\lambda = 1$.

	r=5				r=10			
k	3.530553	3.530554	3.146534	3.146537	3.146534	3.146532	3.53055	3.530579
g	4	4	3	3	3	3	4	4
m	1	2	2.5	3	1	2	2.5	3
c	ARL							
1	370.01	370.01	370.00	370.01	370.00	370.00	370.00	370.06
0.99	355.45	341.46	343.18	338.05	359.03	348.38	334.67	328.07
0.98	341.32	314.87	318.05	308.58	348.27	327.82	302.41	290.50
0.97	301.47	245.64	252.01	233.38	317.30	272.12	221.74	200.20
0.95	276.91	207.27	214.91	192.80	297.71	239.56	179.33	155.20
0.93	242.93	159.56	168.16	143.66	269.87	196.87	129.34	104.88
0.9	193.39	101.19	109.71	86.09	227.42	139.86	73.25	53.07
0.85	151.85	62.49	69.81	50.10	189.69	97.36	40.18	25.90
0.8	117.42	37.49	43.22	28.25	156.38	66.26	21.31	12.21
0.75	89.22	21.81	25.96	15.42	127.24	43.97	10.95	5.63
0.7	48.39	6.72	8.52	4.25	80.31	17.74	2.77	1.41
0.6	23.60	2.00	2.58	1.33	46.69	6.29	1.07	1.00
0.5	9.97	1.02	1.08	1.00	24.15	2.07	1.00	1.00
0.3	3.52	1.00	1.00	1.00	10.48	1.03	1.00	1.00
0.2	1.23	1.00	1.00	1.00	3.48	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.07	1.00	1.00	1.00

and variance

$$\sigma_{v^*} = \frac{(r\lambda^m)^{2/3}\,\Gamma(g+2/3)}{\Gamma(g)} - (\mu_{v^*})^2 \tag{3}$$

Therefore, the control limits for the proposed control chart are given as

$$LCL = \frac{\left(r\lambda_0^m\right)^{1/3}\Gamma(g+1/3)}{\Gamma(g)} - k\sqrt{\frac{\left(r\lambda_0^m\right)^{2/3}\Gamma(q+2/3)}{\Gamma(q)} - \mu_{v^*}^2} \tag{4}$$

$$UCL = \frac{\left(r\lambda_0^m\right)^{1/3}\Gamma(g+1/3)}{\Gamma(g)} + k\sqrt{\frac{\left(r\lambda_0^m\right)^{2/3}\Gamma(g+2/3)}{\Gamma(g)} - \mu_{v^{*2}}} \tag{5}$$

where λ_0 is the scale parameter when the process is in control.

For the proposed control chart, the process is declared to be out-of-control if $v^*\langle LCL$ or $v^*\rangle UCL$. So, the probability that the process is declared as out-of-control when the process is actually in control is given as follows:

$$P_{out}^0 = P\{v^* < LCL|\lambda_0\} + P\{v^* > UCL|\lambda_0\} \tag{6}$$

TABLE 5. The values of ARL when $ARL_0 = 300$; $\lambda = 2$.

	r=5				r=10			
k	3.102293	3.10229	3.10229	3.489704	3.4897	3.4897	3.489747	3.489962
g	3	3	3	4	4	4	4	4
m	1	2	2.5	3	1	2	2.5	3
c	ARL							
1	300.01	300.00	300.00	300.01	300.00	300.00	300.07	300.40
0.99	291.11	282.47	278.26	265.98	288.20	276.86	271.43	266.33
0.98	282.39	265.81	257.89	235.53	276.75	255.30	245.27	235.84
0.97	257.29	220.66	204.36	162.34	244.45	199.19	179.87	162.55
0.95	241.41	194.27	174.29	125.87	224.54	168.10	145.49	126.04
0.93	218.84	159.67	136.40	85.09	197.00	129.43	104.96	85.20
0.9	184.43	113.46	89.02	43.10	156.84	82.11	59.48	43.16
0.85	153.85	79.01	56.68	21.09	123.18	50.75	32.67	21.11
0.8	126.85	53.81	35.12	10.00	95.26	30.49	17.37	10.01
0.75	103.23	35.74	21.14	4.67	72.41	17.77	8.97	4.67
0.7	65.19	14.48	7.01	1.28	39.32	5.55	2.36	1.28
0.6	37.94	5.20	2.20	1.00	19.22	1.74	1.03	1.00
0.5	19.67	1.79	1.04	1.00	8.18	1.01	1.00	1.00
0.3	8.60	1.01	1.00	1.00	2.96	1.00	1.00	1.00
0.2	2.93	1.00	1.00	1.00	1.14	1.00	1.00	1.00
0.1	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 6. The values of ARL when $ARL_0 = 370$; $\lambda = 2$.

	r=5				r=10			
k	3.146533	3.146534	3.530552	3.146536	3.530549	3.530555	3.146624	3.530549
g	3	3	4	3	4	4	3	4
m	1	2	2.5	3	1	2	2.5	3
c	ARL							
1	370.00	370.01	370.01	370.01	370.00	370.01	370.16	370.00
0.99	359.03	348.38	334.68	338.05	355.44	341.47	343.32	328.02
0.98	348.27	327.82	302.41	308.58	341.32	314.87	318.19	290.45
0.97	317.30	272.12	221.74	233.38	301.46	245.64	252.11	200.16
0.95	297.71	239.57	179.34	192.80	276.91	207.27	215.00	155.17
0.93	269.87	196.87	129.34	143.66	242.93	159.56	168.24	104.86
0.9	227.42	139.86	73.25	86.09	193.38	101.19	109.76	53.06
0.85	189.69	97.36	40.18	50.10	151.85	62.50	69.84	25.90
0.8	156.38	66.27	21.31	28.25	117.41	37.50	43.24	12.21
0.75	127.24	43.97	10.95	15.42	89.22	21.81	25.97	5.63
0.7	80.31	17.74	2.77	4.25	48.39	6.72	8.53	1.41
0.6	46.69	6.29	1.07	1.33	23.60	2.00	2.58	1.00
0.5	24.15	2.07	1.00	1.00	9.97	1.02	1.08	1.00
0.3	10.48	1.03	1.00	1.00	3.52	1.00	1.00	1.00
0.2	3.48	1.00	1.00	1.00	1.23	1.00	1.00	1.00
0.1	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00

It is rewritten by

$$P_{out}^{0} = P\left\{v^{\frac{1}{3}} < LCL\right\} + P\left\{v^{\frac{1}{3}} > UCL\right\}$$
$$= P\left\{v < LCL^{3}\right\} + 1 - P\left\{v < UCL^{3}\right\} \quad (7)$$

Here,

$$P\left(v \le LCL^{3}\right) = \sum_{j=1}^{g-1} \frac{e^{-\frac{LCL^{3}}{r\lambda^{m}}}\left(LCL^{3}/r\lambda^{m}\right)^{j}}{j!}$$

and

$$P\left\{v < UCL^{3}\right\} = \sum_{j=1}^{g-1} \frac{e^{-\frac{UCL^{3}}{r\lambda^{m}}}\left(UCL^{3}/r\lambda^{m}\right)^{j}}{j!}$$

Therefore, P_{out}^{0} can be obtained by

$$P_{out}^{0} = 1 - \sum_{j=1}^{g-1} \frac{e^{-LCL^{3}/r\lambda_{0}^{m}}\left(LCL^{3}/r\lambda_{0}^{m}\right)^{j}}{j!} + \sum_{j=1}^{g-1} \frac{e^{-UCL^{3}/r\lambda_{0}^{m}}\left(UCL^{3}/r\lambda_{0}^{m}\right)^{j}}{j!} \quad (8)$$

The performance of a control chart is usually evaluated by the average run length (ARL). The in-control ARL is given by (9), shown at the bottom of the next page. Suppose now that the scale parameter is shifted to $\lambda_1 = c\lambda_0$. Then, the probability (P_{out}^{1}) of the process being declared to be out-of-control when the process is shifted is:

$$P^1_{out} = P\{v^* < LCL|\lambda_1 = c\lambda_0\} + P\{v^* > UCL|\lambda_1 = c\lambda_0\} \quad (10)$$

$$P^1_{out} = 1 - \sum_{j=1}^{g-1} \frac{e^{-LCL^3/r\lambda_1^m}(LCL^3/r\lambda_1^m)^j}{j!} + \sum_{j=1}^{g-1} \frac{e^{-UCL^3/r\lambda_1^m}(UCL^3/r\lambda_1^m)^j}{j!} \quad (11)$$

The out-of-control ARL for the shifted process is given as (12), shown at the bottom of this page. The values of ARL_1 for various values of specified parameters are presented in Tables 1-6. We fixed two values of number of testers $r = 5, 10$, shift c and ARLs. The values of g and m are determined. Table 1-2 are presented for various shift c, $\lambda = 0.5$ and specified ARL_0, say r_0 is 300 and 370. Tables 3-4 are presented for $\lambda = 1$ and ARL_0 is 300 and 370. Tables 5-6 are placed for $\lambda = 2$ and ARL_0 is 300 and 370.

From Tables 1-6, we note following trends in control chart parameters.

1) For same values of all other parameters, ARL_1 decreases as λ increases from 0.5 to 2.
2) For same values of all other parameters, ARL_1 increases as m increases from 1 to 3.
3) For same values of all other parameters, ARL_1 decreases as r increases from 5 to 10.

The following algorithm is used to find ARL_1, k and g.

1) Specify $ARL_0 = 300, 370$; $r = 5,10$; and m.
2) Determine k such that $ARL_0 \geq ARL_0$
3) Find ARL_1 according to the values of c for selected k

III. APPLICATION OF PROPOSED CHART

In this section, we will discuss the application of the proposed control chart using an artificial data and a real data obtained from a bearing manufacturing company.

A. ARTIFICIAL DATA

In this section, we will show the performance of proposed control chart using simulated data. For this simulation study, we assume that $\lambda = 2$, $m = 2$, $r = 5$ and $g = 3$. The data Y_i's are generated from the Weibull distribution with shape parameter m and scale parameter $\lambda r^{1/m}$ when the process is in control and the data Y_i's are generated from the Weibull distribution with shape parameter m and scale parameter $0.7\lambda r^{1/m}$ when the process is shifted. First 20 values are generated from in-control process and next 20 observations from shifted process. The simulated data is shown in Table 7. The statistic v^* is computed and shown in Table 7.

The control limits from the in-control data are obtained by $LCL = 1.4073$ and $UCL = 6.1338$. The values of v^* are plotted on the proposed control chart in Figure 1.

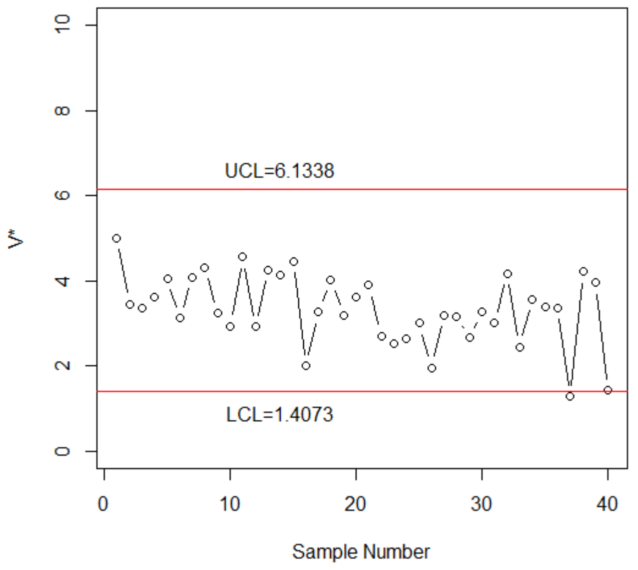

FIGURE 1. The control chart for simulated data.

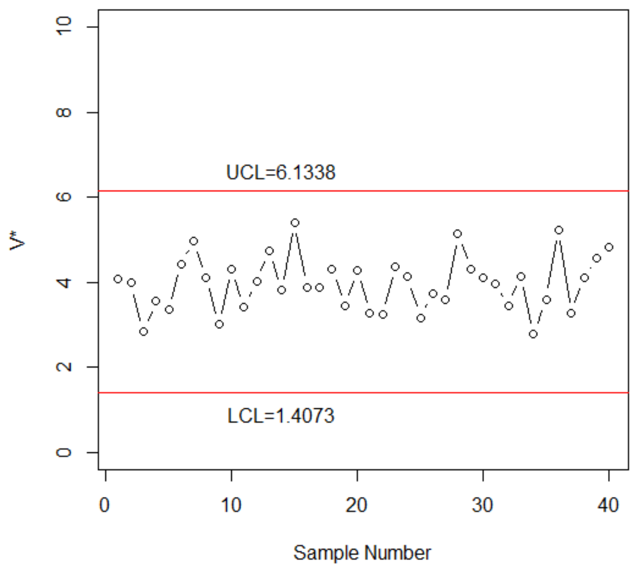

FIGURE 2. The proposed chart for real data.

$$ARL_0 = \frac{1}{1 - \sum_{j=1}^{g-1} \frac{e^{-LCL^3/r\lambda_0^m}(LCL^3/r\lambda_0^m)^j}{j!} + \sum_{j=1}^{g-1} \frac{e^{-UCL^3/r\lambda_0^m}(UCL^3/r\lambda_0^m)^j}{j!}} \quad (9)$$

$$ARL_1 = \frac{1}{1 - \sum_{j=1}^{g-1} \frac{e^{-LCL^3/r\lambda_1^m}(LCL^3/r\lambda_1^m)^j}{j!} + \sum_{j=1}^{g-1} \frac{e^{-UCL^3/r\lambda_1^m}(UCL^3/r\lambda_1^m)^j}{j!}} \quad (12)$$

TABLE 7. The simulated data.

Sr#	Yi's			
	1	2	3	v*
1	9.521185	3.687743	4.607741	5.006441
2	1.865519	4.038575	4.614244	3.4505
3	1.431465	3.385011	4.912179	3.351233
4	1.135338	6.547314	1.65866	3.606456
5	6.492342	4.528849	1.899786	4.046747
6	4.147889	1.73047	3.241334	3.13141
7	4.139434	3.653806	6.161775	4.090692
8	7.393612	4.565883	2.144854	4.3109
9	4.143483	3.471952	2.179491	3.238757
10	1.973474	4.609762	0.445603	2.937332
11	7.670984	5.118874	3.047797	4.552246
12	3.731001	2.005349	2.601238	2.912598
13	4.195149	6.577462	3.985337	4.249623
14	3.734449	5.846805	4.743502	4.133652
15	3.998439	8.066407	2.709039	4.454577
16	1.589059	0.280453	2.327163	2.00162
17	2.86003	3.038659	4.171112	3.26518
18	2.851136	6.084647	4.495552	4.028174
19	5.203626	0.577831	2.245158	3.189692
20	3.846271	4.746208	3.07018	3.602321
21	3.783086	0.542585	6.66767	3.894403
22	1.344575	2.36778	3.527983	2.708111
23	1.257962	2.762949	2.638212	2.529075
24	1.342806	3.891105	1.257952	2.646038
25	2.40425	3.543137	2.963193	3.004244
26	0.959815	1.212899	2.25691	1.956216
27	3.858921	3.515457	2.238231	3.183357
28	2.711807	4.52549	1.901871	3.156544
29	2.395621	3.493858	0.856643	2.653331
30	1.285116	3.89909	4.208398	3.25746
31	1.329027	2.882516	4.149054	3.010697
32	4.669327	4.185677	5.753682	4.168382
33	0.914754	2.525036	2.726456	2.446665
34	1.625006	5.140723	3.974389	3.553292
35	4.665399	1.846819	3.698544	3.38703
36	2.605654	4.404121	3.469738	3.368592
37	0.293305	0.809499	1.156123	1.276079
38	2.591531	2.40597	7.915057	4.220026
39	1.560297	4.123714	6.488563	3.9481
40	0.983747	1.037736	0.921615	1.425064

From Figure 1, it can be noted that the proposed control chart detects shift at 37th sample, which is the 17th sample after the process shift. So, the proposed control chart has efficiency to detect shift in the process.

TABLE 8. The data for ball bearing.

Sr#	Yi's			
	1	2	3	v*
1	2.009385	7.779006	1.752351	4.074064
2	5.960827	3.217039	4.13765	3.979077
3	1.729362	4.127599	1.659021	2.834775
4	3.609626	4.108306	3.892758	3.558504
5	5.054601	1.877135	2.928153	3.351524
6	4.426042	5.617833	5.985856	4.430714
7	6.123741	6.243373	6.859577	4.980372
8	6.536124	2.37621	4.574555	4.10738
9	1.566379	3.114121	3.858613	3.001488
10	4.773247	2.235248	7.252549	4.315675
11	4.62858	3.478084	2.52234	3.416615
12	3.875296	4.26413	5.612815	4.014622
13	6.241229	5.970449	5.67346	4.744312
14	6.863653	1.599389	2.34034	3.806291
15	8.844831	7.478871	4.875911	5.405425
16	3.452253	4.972192	4.659262	3.878635
17	5.611566	0.507887	5.083183	3.861653
18	3.447337	3.134791	7.592218	4.297219
19	1.695614	5.297071	3.13846	3.442151
20	3.449106	5.70535	5.801958	4.274668
21	2.74742	4.989276	1.635562	3.274684
22	3.108467	4.727357	1.328097	3.232428
23	3.457512	6.644438	5.202562	4.36504
24	2.612608	3.234709	7.259367	4.121039
25	0.704539	5.320732	1.704934	3.165294
26	3.129609	4.061275	5.037083	3.724373
27	3.484034	5.70558	1.066568	3.578621
28	10.80973	2.10554	3.781144	5.137272
29	2.920218	7.199859	4.328848	4.292733
30	5.867873	2.070135	5.462089	4.092666
31	7.426672	1.852443	1.965285	3.96743
32	3.72919	1.474369	4.984183	3.446049
33	6.281662	5.382718	1.688685	4.146343
34	0.947883	4.323917	1.340808	2.776007
35	4.090849	4.059047	3.60756	3.588891
36	5.886786	5.752082	8.687733	5.23197
37	4.255701	3.032014	2.759001	3.268453
38	6.916949	2.316732	4.041306	4.112308
39	3.300406	9.049077	1.796128	4.578929
40	3.563152	9.971883	0.897838	4.833741

B. REAL DATA

Now, we apply the proposed control chart to a real data obtained from a bearing manufacturing company. The similar example was given by [20] Suppose that quality assurance department of this company decides to monitor the reliability of their product using the proposed control chart with the target in-control ARL of 370. They use sudden death testing with $r = 5$ and $g = 3$. The data is well fitted to the Weibull distribution with $m = 2$. The first failure time from each group is noted and the data is reported in Table 8. The values of statistic v^* are computed and placed in Table 8.

The values of statistics v^* are plotted on Figure 2. It can be seen that all plotted values are within the control limits which indicate that the process of ball bearing is in-control.

IV. CONCLUSIONS

A variable control chart for monitoring reliability of a product under sudden death life testing is proposed under the assumption that the lifetime of the product follows a Weibull distribution. A power transform is utilized so as to design symmetric type control limits. The in-control ARL and the ARL for a shifted process are derived. The simulation study shows that the proposed control chart has ability to detect a shift in the process. An example is also given to illustrate the use of the proposed control chart in the industry. As Weibull distribution may be also asymmetric depending on the values of the parameters, it could be very interesting in determining the control limits for the gamma distribution and in comparing the performance with asymmetric versus symmetric distributions as future research.

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