1/2-BPS D-branes from covariant open superstring in AdS$_4 \times$ CP^3 background

Jaemo Park[a] and Hyeonjoon Shin[a,b]

[a]*Department of Physics & Center for Theoretical Physics,*
 POSTECH, Pohang, Gyeongbuk 37673, South Korea

[b]*Asia Pacific Center for Theoretical Physics,*
 Pohang, Gyeongbuk 37673, South Korea

 E-mail: jaemo@postech.ac.kr, nonchiral@gmail.com

ABSTRACT: We consider the open superstring action in the AdS$_4 \times$ **CP**^{3} background and investigate the suitable boundary conditions for the open superstring describing the 1/2-BPS D-branes by imposing the κ-symmetry of the action. This results in the classification of 1/2-BPS D-branes from covariant open superstring. It is shown that the 1/2-BPS D-brane configurations are restricted considerably by the Kähler structure on **CP**^{3}. We just consider D-branes without worldvolume fluxes.

Contents

1 Introduction

It has been proposed that the Type IIA string theory on the $AdS_4 \times \mathbf{CP}^3$ background is dual to the three-dimensional superconformal $\mathcal{N} = 6$ Chern-Simons theory with gauge group $U(N)_k \times U(N)_{-k}$ known as the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [1]. To be more precise, since the ABJM theory is motivated by the description of multiple M2-branes, it is dual to the M-theory on $AdS_4 \times S^7/\mathbf{Z}_k$ geometry with N units of four-form flux turned on AdS_4, where N and k correspond to the rank of the gauge group and the integer Chern-Simons level respectively. When $1 \ll N^{1/5} \ll k \ll N$, the M-theory can be dimensionally reduced to the Type IIA string theory on the $AdS_4 \times \mathbf{CP}^3$ background.

After the proposal of this new type of duality, various supersymmetric embeddings of D-branes have been considered. Embeddings for the giant graviton [2–10], adding flavor [11–14], and some other purposes [15, 16] are the examples studied extensively. With some other motivations, we may also consider other types of supersymmetric D-brane embeddings or configurations. Since each of them would correspond to a specific object in the dual gauge theory, the exploration of supersymmetric D-branes may be regarded as an important subject to enhance our understanding of duality. However, unlike the case of flat spacetime, the sturucture of $AdS_4 \times \mathbf{CP}^3$ background is not so trivial and the solution of the associated Killing spinor equation is rather complicated. This makes the case by case study of supersymmetric D-branes laborious, and thus it seems to be desirable to have some guideline. In this paper, we focus especially on the most supersymmetric cases and are trying to classify the 1/2-BPS D-branes in the $AdS_4 \times \mathbf{CP}^3$ background. In doing so, we are aiming at obtaining the classification data as a guideline for further exploration of supersymmetric D-branes.

For the classification of D-branes, we use the covariant open superstring description, which is especially useful in classifying the 1/2-BPS D-branes. It has been developed in [17] for the flat spacetime background, and successfully applied to some important backgrounds in superstring theory [18–23]. To carry out such classification, we need the Type IIA superstring action in the $AdS_4 \times \mathbf{CP}^3$ background, which has been constructed by using the super coset structure [24–27]. However, the action is the one where the κ-symmetry is

partially fixed, and might be inadequate in describing all possible motions of the string as already pointed out in [24]. The fully κ-symmetric complete action has been constructed in [28], which we take in this paper.

In the next section, we consider the Wess-Zumino (WZ) term of the complete superstring action in the AdS$_4 \times \mathbf{CP}^3$ background, which is the ingredient for the covariant open string description of 1/2-BPS D-branes, and set our notation and convention. In section 3, we investigate the suitable boundary conditions for open string in a way to keep the κ-symmetry and classify the 1/2-BPS D-branes. The discussion with some comments follows in section 4.

2 Wess-Zumino term

The original formulation for the covariant description of D-branes [17] considers an arbitrary variation of the open superstring action and looks for suitable open string boundary conditions to make the action invariant. However, it has been pointed out in [18] that the κ-symmetry is enough at least for the description of supersymmetric D-branes. The basic reason is that the κ-symmetry is crucial for matching the dynamical degrees of freedom for bosons and fermions on the string worldsheet and hence ensuring the object described by the open string supersymmetric.

The κ-symmetry transformation rules in superspace are[1]

$$\delta_\kappa Z^M \mathcal{E}_M^A = 0 \,, \quad \delta_\kappa Z^M \mathcal{E}_M = \frac{1}{2}(1 + \Gamma)\kappa \,, \tag{2.1}$$

where $Z^M = (X^\mu, \Theta)$ is the supercoordinate, $\mathcal{E}_M^A$ ($\mathcal{E}_M$) is the vector (spinor) superfield,[2] κ is the 32 component κ-symmetry transformation parameter, and Γ is basically the pullback of the antisymmetric product of two Dirac gamma matrices onto the string worldsheet with the properties, $\Gamma^2 = 1$ and $\mathrm{Tr}\,\Gamma = 0$, whose detailed expression is not needed here. By construction, the bulk part of the superstring action is invariant under this κ-symmetry transformation. In the case of open superstring, however, we have non-vanishing contributions from the worldsheet boundary, the boundary contributions, under the κ-symmetry variation. Interestingly, as noted in [18], the kinetic part of the superstring action does not give any boundary contribution due to the first equation of (2.1). Thus, only the WZ term rather than the full superstring action is of our concern in considering the boundary contributions.

---

[1]The notation and convention for indices are as follows. The spinor index for the fermionic object is that of Majorana spinor having 32 real components and suppressed as long as there is no confusion. μ is the ten-dimensional curved space-time vector index. As for the Lorentz frame or the tangent space, the vector index is denoted by

$$A = (a, a') \,, \quad a = 0, 1, 2, 3 \,, \quad a' = 1', \dots, 6' \,,$$

where a (a') corresponds to the tangent space of AdS$_4$ ($\mathbf{CP}^3$), and the metric η_{AB} follows the most plus sign convention as $\eta_{AB} = \mathrm{diag}(-, +, +, \dots, +)$.

[2]In the present case, $\mathcal{E}_M^A$ and $\mathcal{E}_M$ are of course the superfields for the AdS$_4 \times \mathbf{CP}^3$ background whose explicit expressions have been derived in [28].

The WZ term has an expansion in terms of the fermionic coordinate Θ up to the order of Θ^{32}. Here, we will consider the expansion up to quartic order. From the complete Type IIA superstring action in the $\text{AdS}_4 \times \mathbf{CP}^3$ background [28], we see that the expansion of the WZ term has the following form.

$$S_{\text{WZ}} = S^{(2)} + S^{(4)} + \mathcal{O}(\Theta^6)\,, \tag{2.2}$$

where $S^{(2)}$ and $S^{(4)}$ represent the quadratic and quartic part respectively.

The quadratic part is read off as[3]

$$S^{(2)} = \frac{R}{k} \int_\Sigma \left[i e^A \wedge \Theta \Gamma_A \Gamma_{11} D\Theta - \frac{1}{R} e^b \wedge e^a \left(\chi \gamma_{ab} \gamma^7 \chi \right) \right.$$
$$\left. - \frac{1}{R} e^{b'} \wedge e^{a'} (\Theta \gamma_{a'b'} \gamma^7 \chi) - \frac{2}{R} e^{a'} \wedge e^a (\theta \gamma_a \gamma_{a'} \gamma^5 \gamma^7 \chi) \right]\,, \tag{2.3}$$

where Σ is the open string worldsheet. The $\text{AdS}_4 \times \mathbf{CP}^3$ background is obtained by the dimensional reduction of the eleven dimensional $\text{AdS}_4 \times \text{S}^7/\mathbf{Z}_k$ background. This gives the origin of the appearance of k in the action. R is the radius of S^7 in the eleven dimensional Planck unit and has the relation with the $\mathbf{CP}^3$ radius, $R_{\mathbf{CP}^3}$ in string unit, as $R^2_{\mathbf{CP}^3} = R^3/k = 4\pi\sqrt{2N/k}$. The radius of AdS_4 is half of $R_{\mathbf{CP}^3}$. The ten dimensional gamma matrices Γ_A are represented through the tensor product of four and six dimensional gamma matrices as

$$\Gamma^a = \gamma^a \otimes 1\,, \quad \Gamma^{a'} = \gamma^5 \otimes \gamma^{a'}\,, \quad \Gamma_{11} = \gamma^5 \otimes \gamma^7\,, \tag{2.4}$$

where Γ_{11} measures the ten dimensional chirality and

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,, \quad \gamma^7 = i\gamma^{1'}\gamma^{2'}\ldots\gamma^{6'}\,. \tag{2.5}$$

The ten dimensional Weyl spinor Θ with 32 real components can be split into two parts in a way to respect the supersymmetry structure of the $\text{AdS}_4 \times \mathbf{CP}^3$ background as

$$\theta = \mathcal{P}_6 \Theta\,, \quad \chi = \mathcal{P}_2 \Theta\,, \tag{2.6}$$

where $\mathcal{P}_6$ and $\mathcal{P}_2$ are the projectors defined by

$$\mathcal{P}_6 = \frac{1}{8}(6 - J)\,, \quad \mathcal{P}_2 = \frac{1}{8}(2 + J)\,, \quad \mathcal{P}_6 + \mathcal{P}_2 = 1\,, \tag{2.7}$$

and J is a quantity depending on the Kähler form $\frac{1}{2}J_{a'b'}e^{a'} \wedge e^{b'}$ on $\mathbf{CP}^3$,

$$J = -iJ_{a'b'}\gamma^{a'b'}\gamma^7\,. \tag{2.8}$$

Because J satisfies $J^2 = 4J + 12$ and hence has six eigenvalues -2 and two eigenvalues 6, θ (χ) has 24 (8) independent components after taking into account the spinorial structure

---

[3]In the practical calculation, we utilize the expressions of superfields given in [29], a subsequent paper after [28]. We mostly follow the notation and convention of [28, 29]. As an exception, we use χ rather than υ to represent spinor components corresponding to the eight broken supersymmetries of the $\text{AdS}_4 \times \mathbf{CP}^3$ background.

in the AdS$_4$ subspace. The spinor χ corresponds to the eight supersymmetries broken by the AdS$_4 \times \mathbf{CP}^3$ background.

The covariant derivative for Θ in (2.3) is defined as

$$D\Theta = (D_{24}\theta, D_8\chi),\tag{2.9}$$

where

$$D_{24}\theta = \mathcal{P}_6 \left(d + \frac{i}{R}e^a\gamma^5\gamma_a + \frac{i}{R}e^{a'}\gamma_{a'} - \frac{1}{4}\omega^{ab}\gamma_{ab} - \frac{1}{4}\omega^{a'b'}\gamma_{a'b'} \right)\theta$$

$$D_8\chi = \mathcal{P}_2 \left(d + \frac{i}{R}e^a\gamma^5\gamma_a - \frac{1}{4}\omega^{ab}\gamma_{ab} - 2iA\gamma^7 \right)\chi\tag{2.10}$$

We would like to note that D_{24} and D_8 can be written as

$$D_{24} = \mathcal{P}_6\mathcal{D}\mathcal{P}_6, \quad D_8 = \mathcal{P}_2\mathcal{D}\mathcal{P}_2,\tag{2.11}$$

where

$$\mathcal{D} = d + \frac{i}{R}e^a\gamma^5\gamma_a + \frac{i}{R}e^{a'}\gamma_{a'} - \frac{1}{4}\omega^{ab}\gamma_{ab} - \frac{1}{4}\omega^{a'b'}\gamma_{a'b'}.\tag{2.12}$$

From this, we see that the Ramond-Ramond one-form gauge potential A in (2.10) has the following expression

$$A = \frac{1}{8}J_{a'b'}\omega^{a'b'}\tag{2.13}$$

through an identity $\mathcal{P}_2\gamma_{a'b'}\mathcal{P}_2 = \frac{i}{6}J_{a'b'}\mathcal{P}_2J\gamma^7\mathcal{P}_2 = iJ_{a'b'}\mathcal{P}_2\gamma^7\mathcal{P}_2$.[4]

If we now move on to the quartic part $S^{(4)}$ in the expansion of the WZ term (2.2), it is read off as

$$\begin{aligned}
S^{(4)} =& \frac{R}{2k}\int_\Sigma \Bigg\{ (\chi\gamma^{a'}\gamma^5\theta)(D\Theta \wedge \gamma_{a'}\gamma^7 D\Theta) \\
& - (\Theta\gamma^a D\Theta) \wedge (\Theta\gamma_a\gamma^5\gamma^7 D\Theta) - \left(\theta\gamma^{a'}\gamma^5 D_{24}\theta + 2\chi\gamma^{a'}\gamma^5 D_{24}\theta\right) \wedge (\Theta\gamma_{a'}\gamma^7 D\Theta) \\
& + \frac{i}{R}e^a \wedge \Bigg[-2(\chi\gamma^5\chi)(\Theta\gamma_a\gamma^5\gamma^7 D\Theta) - 2(\chi\gamma^b\gamma^7\chi)(\Theta\gamma_{ab}D\Theta) + 2(\chi\gamma_a\gamma^5\chi)(\Theta\Gamma_{11}D\Theta) \\
& \quad - 4(D_{24}\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi)(\chi\gamma^{a'}\gamma^5\theta) + \left(\theta\gamma^b D_{24}\theta + 2\chi\gamma^b D_8\chi\right)(\chi\gamma_{ab}\gamma^7\chi) \\
& \quad + 2\left(\theta\gamma^{a'}\gamma^5 D_{24}\theta + 2\chi\gamma^{a'}\gamma^5 D_{24}\theta\right)(\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi)\Bigg] \\
& + \frac{i}{R}e^{a'} \wedge \Bigg[-2(\chi\gamma^5\chi)(\Theta\gamma_{a'}\gamma^7 D\Theta) + 2(\chi\gamma^a\gamma^7\chi)(\Theta\gamma_a\gamma_{a'}\gamma^5 D\Theta) + 2(\theta\gamma_{a'}\chi)(\Theta\Gamma_{11}D\Theta) \\
& \quad - 4(D\Theta\gamma_{a'b'}\gamma^7\chi)(\chi\gamma^{b'}\gamma^5\theta) + 2\left(\theta\gamma^{b'}\gamma^5 D_{24}\theta + 2\chi\gamma^{b'}\gamma^5 D_{24}\theta\right)(\Theta\gamma_{a'b'}\gamma^7\chi) \\
& \quad - (\theta\gamma^a D_{24}\theta + 2\chi\gamma^a D_8\chi)(\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi) - \frac{1}{2}(\theta\gamma^{ab}\gamma^5 D_{24}\theta)(\theta\gamma_{ab}\gamma_{a'}\gamma^7\chi)\Bigg] \\
& + \frac{i}{6}e^a \wedge \left(\theta\gamma_a\gamma^5\gamma^7\mathcal{M}^2 D_{24}\theta - D_{24}\theta\gamma_a\gamma^5\gamma^7\mathcal{M}^2\theta + \chi\gamma_a\gamma^5\gamma^7\mathcal{W}^2 D_8\chi\right) \\
& + \frac{i}{6}e^{a'} \wedge \left(\Theta\gamma_{a'}\gamma^7\mathcal{M}^2 D_{24}\theta - D\Theta\gamma_{a'}\gamma^7\mathcal{M}^2\theta + \theta\gamma_{a'}\gamma^7\mathcal{W}^2 D_8\chi\right) + \ldots \Bigg\},
\end{aligned}\tag{2.14}$$

---

[4]See eq. (C.31) in [28].

where $\mathcal{M}^2$ and $\mathcal{W}^2$ are defined as

$$
\begin{aligned}
R(\mathcal{M}^2)^{\alpha\alpha'}{}_{\beta\beta'} = {}& 4\theta^\alpha_{\beta'}(\theta^{\alpha'}\gamma^5)_\beta - 4\delta^{\alpha'}_{\beta'}\theta^{\alpha\sigma'}(\theta\gamma^5)_{\beta\sigma'} \\
& - 2(\gamma^5\gamma^a\theta)^{\alpha\alpha'}(\theta\gamma_a)_{\beta\beta'} - (\gamma^{ab}\theta)^{\alpha\alpha'}(\theta\gamma_{ab}\gamma^5)_{\beta\beta'}\,, \\
R(\mathcal{W}^2)^{\alpha i}{}_{\beta j} = {}& -4(\gamma^7\chi)^{\alpha i}(\chi\gamma^7\gamma^5)_{\beta j} - 2(\gamma^5\gamma^a\chi)^{\alpha i}(\chi\gamma_a)_{\beta j} - (\gamma^{ab}\chi)^{\alpha i}(\chi\gamma_{ab}\gamma^5)_{\beta j}\,. \quad (2.15)
\end{aligned}
$$

The dots in the last line denote the terms which lead to the boundary contributions of higher order in Θ (Θ^5 order) under the κ symmetry transformation and hence should be considered together with the transformation of sextic oder part of the WZ term.

3 Covariant description of 1/2-BPS D-branes

In this section, we take the κ-symmetry variation of the WZ term considered in the previous section and obtain the boundary contributions. We then investigate the suitable open string boundary conditions which make the boundary contributions vanish and hence guarantee the κ-symmetry, the boundary κ-symmetry. The resulting open string boundary conditions give the covariant description of 1/2-BPS D-branes.

In taking the κ-symmetry variation, it is convenient to express the variation of X^μ in terms of $\delta_\kappa\Theta$ by using the first equation of (2.1) as

$$
\delta_\kappa X^\mu = -i\Theta\Gamma^\mu\delta_\kappa\Theta + \mathcal{O}(\Theta^3)\,, \quad (3.1)
$$

where we retain the variations up to the quadratic order in Θ because we are interested in the κ-symmetry variation of the WZ term up to the quartic order in Θ. By exploiting this, we first consider the boundary contributions from the κ-symmetry variation of quadratic part independent of the spin connection, which are as follows:

$$
\begin{aligned}
\delta_\kappa S^{(2)} \longrightarrow i\frac{R}{k}\int_{\partial\Sigma}\Big[& -e^A\Theta\Gamma_A\Gamma_{11}\delta_\kappa\Theta - i(\Theta\Gamma^A\delta_\kappa\Theta)(\Theta\Gamma_A\Gamma_{11}d\Theta) \\
& +\frac{2}{R}e^a(\Theta\gamma^b\delta_\kappa\Theta)(\theta\gamma_{ab}\gamma^7\theta + 2\chi\gamma_{ab}\gamma^7\chi) + \frac{2}{R}e^{a'}(\Theta\gamma^{b'}\gamma^5\delta_\kappa\Theta)(\Theta\gamma_{a'b'}\gamma^7\Theta) \\
& +\frac{2}{R}e^a(\Theta\gamma^{a'}\gamma^5\delta_\kappa\Theta)(\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi) - \frac{2}{R}e^{a'}(\Theta\gamma^a\delta_\kappa\Theta)(\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi)\Big]\,, \quad (3.2)
\end{aligned}
$$

where $\partial\Sigma$ represents the boundary of open string worldsheet Σ. For the boundary κ-symmetry, each term should vanish under a suitable set of open string boundary conditions.

Let us look at the first term. Because

$$
dX^\mu e^A_\mu = 0 \quad (A \in D)\,, \quad (3.3)
$$

where $A \in D$ (N) implies that A is a Dirichlet (Neumann) direction, the fermion bilinear $\Theta\Gamma_A\Gamma_{11}\delta_\kappa\Theta$ should vanish for $A \in N$. In order to check this at the worldsheet boundary, we firstly split the ten dimensional Majorana spinor Θ into two Majorana-Weyl spinors Θ^1 and Θ^2 with opposite ten diemensional chiralities as

$$
\Theta = \Theta^1 + \Theta^2\,, \quad (3.4)
$$

where we take $\Gamma_{11}\Theta^1 = \Theta^1$ and $\Gamma_{11}\Theta^2 = -\Theta^2$. Secondly, we impose the following boundary condition breaking the background supersymmetry by half

$$\Theta^2 = P\Theta^1 \qquad (3.5)$$

with

$$P = s\Gamma^{A_1\ldots A_{p+1}}\,, \qquad (3.6)$$

where all the indices $A_1,\ldots,A_{p+1}$ are those for Neumann directions, and

$$s = \begin{cases} 1 \text{ for } X^0 \in N \\ i \text{ for } X^0 \in D \end{cases}, \qquad (3.7)$$

depending on the boundary condition for the time direction X^0. It should be noted that p must be even because Θ^1 and Θ^2 have opposite chiralities. Then $\Theta^2\Gamma_A\delta_\kappa\Theta^2$ is evaluated to be $\Theta^1\Gamma_A\delta_\kappa\Theta^1$ for $A \in N$ or $-\Theta^1\Gamma_A\delta_\kappa\Theta^1$ for $A \in D$, which means that

$$\Theta\Gamma_A\Gamma_{11}\delta_\kappa\Theta = \Theta^1\Gamma_A\delta_\kappa\Theta^1 - \Theta^2\Gamma_A\delta_\kappa\Theta^2 = 0 \quad (A \in N)\,,$$
$$\Theta\Gamma_A\delta_\kappa\Theta = \Theta^1\Gamma_A\delta_\kappa\Theta^1 + \Theta^2\Gamma_A\delta_\kappa\Theta^2 = 0 \quad (A \in D)\,. \qquad (3.8)$$

The first identity of this equation clearly shows that the first term of (3.2) vanishes under the boundary condition of eq. (3.5). Another consequence of eq. (3.8) is that the second term of (3.2) becomes zero automatically since $\Theta\Gamma_A\Gamma_{11}\delta_\kappa\Theta = 0$ ($A \in N$) also implies $\Theta\Gamma_A\Gamma_{11}d\Theta = 0$ ($A \in N$).

Now we consider the fourth term of (3.2) prior to the third one which requires us some care. From eqs. (3.3) and (3.8), the vanishing condition for the term is

$$\Theta\gamma_{a'b'}\gamma^7\Theta = 0 \quad (a',b' \in N)\,. \qquad (3.9)$$

In order to see when this condition is satisfied, it is convenient to introduce two integers n and n' to denote the number of Neumann directions in AdS_4 and $\mathbf{CP}^3$ respectively. Then we have the relation,

$$n + n' = p + 1\,, \qquad (3.10)$$

and the matrix P of (3.6) for the boundary condition (3.5) is expressed as

$$P = s\Gamma^{a_1\ldots a_n a'_1\ldots a'_{n'}} = s\gamma^{a_1\ldots a_n}(\gamma^5)^{n'} \otimes \gamma^{a'_1\ldots a'_{n'}}\,. \qquad (3.11)$$

A bit of calculation by using this P shows that the condition (3.9) is satisfied for the following cases:

$$(n,n') = (\mathrm{odd},\mathrm{even}) \quad \text{for } p = 0 \bmod 4\,,$$
$$(n,n') = (\mathrm{even},\mathrm{odd}) \quad \text{for } p = 2 \bmod 4\,, \qquad (3.12)$$

according to which the possible candidates of 1/2-BPS Dp-brane are listed as

$$
\begin{aligned}
p = 0 : &\quad (1,0) \\
p = 2 : &\quad (0,3),\ (2,1) \\
p = 4 : &\quad (1,4),\ (3,2) \\
p = 6 : &\quad (2,5),\ (4,3) \\
p = 8 : &\quad (3,6).
\end{aligned}
\tag{3.13}
$$

The first two terms and the fourth term on the right hand side of eq. (3.2) that we have considered are written in terms of the Weyl spinor Θ alone. On the other hand, the third and the last two terms have explicit dependence on χ (θ), the specific part of Θ corresponding to the (un-)broken supersymmetry of AdS$_4 \times \mathbf{CP}^3$ background.

As for the third term, the condition making it vanish is

$$
\theta \gamma_{ab} \gamma^7 \theta = 0, \quad \chi \gamma_{ab} \gamma^7 \chi = 0 \quad (a, b \in N)
\tag{3.14}
$$

due to eqs. (3.3) and (3.8). It is not difficult to check that these conditions are satisfied for the cases of (3.12) if we split θ and χ as (3.4) and if we *can* apply the boundary conditions

$$
\theta^2 = P\theta^1,\ \chi^2 = P\chi^1
\tag{3.15}
$$

similar to (3.5). However, the condition (3.15) is incompatible with (3.5). If we recall the definitions of θ and χ given in eq. (2.6), we see that these boundary conditions (3.15) assume implicitly the commutativity of P with $\mathcal{P}_6$ and $\mathcal{P}_2$, or more basically $[P, J] = 0$ from eq. (2.7). This assumption is too naive because $[P, J] \neq 0$ generically. In fact, if $\mathcal{P}_6$ ($\mathcal{P}_2$) acts on the boundary condition (3.5) and the definition of θ (χ) of eq. (2.6) is used, the correct boundary condition for θ (χ) turns out to be

$$
\begin{aligned}
\theta^2 &= P\theta^1 + \frac{1}{8}[P, J]\Theta^1, \\
\chi^2 &= P\chi^1 - \frac{1}{8}[P, J]\Theta^1.
\end{aligned}
\tag{3.16}
$$

As one may guess, the conditions of (3.14) are not satisfied under these boundary conditions due to Θ^1 dependent terms which do not vanish by themselves. We may introduce additional suitable boundary condition for Θ^1 to get desired situation. However, this leads to lower supersymmetry. Since we are focusing on the 1/2-BPS D-branes, we are not trying to consider such additional boundary condition. Instead we explore the cases in which P commutes with J.

The matrix J depends on the Kähler form $\frac{1}{2}J_{a'b'}e^{a'} \wedge e^{b'}$ on $\mathbf{CP}^3$ as one can see from eq. (2.8). It is convenient to choose a local frame such that the tangent space components $J_{a'b'}$ take the canonical form [30]

$$
J_{a'b'} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\tag{3.17}
$$

Since three two dimensional subspaces are equivalent in this form, it is enough to consider one subspace when investigating the commutativity between P with J. For a given two dimensional subspace, we can now check that $[P, \gamma^{a'b'}\gamma^7] = 0$ when

$$a', b' \in N \text{ or } D \quad (n' = \text{even}),$$
$$a' \in N(D), \ b' \in D(N) \quad (n' = \text{odd}). \tag{3.18}$$

This implies that $[P, J] = 0$ under the following conditions:

1. for even n', both of two directions in each two dimensional subspace are Neumann or Dirichlet one.

2. for odd n', one of two directions in each two dimensional subspace is Neumann one and another is Dirichlet one. This restricts the value of odd n' to 3.

These two conditions make the boundary condition for θ (χ) of eq. (3.16) have the same form with (3.5), and in turn eq. (3.14) is satisfied. They also constrain the configurations of 1/2-BPS D-branes. Especially, the condition (ii) that specifies $n' = 3$ for odd n' informs us that the two D-branes in (3.13)

$$(2, 1), \quad (2, 5) \tag{3.19}$$

are not 1/2-BPS and thus should be excluded from the list of 1/2-BPS D-branes. As a result, we see that the possible configurations of 1/2-BPS D-branes are restricted considerably by the Kähler structure on $\mathbf{CP}^3$.

From eqs. (3.3) and (3.8), we see that the last two terms of (3.2) vanish if

$$\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi = 0 \quad (a, a' \in N). \tag{3.20}$$

It is not difficult to check that this is indeed satisfied for the cases of (3.12) and under the conditions (i) and (ii) of the previous paragraph.

Having investigated the vanishing conditions for the boundary contributions from the quadratic part independent of the spin connection, we now move on to the boundary contributions from the κ-symmetry variation of the spin connection dependent terms. They are obtained as

$$
\begin{aligned}
\delta_\kappa(S^{(2)} + S^{(4)}) \longrightarrow \frac{R}{4k} \int_{\partial\Sigma} \omega_\mu^{ab} \Big\{ &-\frac{1}{2} dX^\mu (\Theta\gamma^c\delta_\kappa\Theta)(\Theta\gamma_c\gamma_{ab}\gamma^5\gamma^7\Theta) \\
&- dX^\mu (\Theta\gamma^{c'}\gamma^5\delta_\kappa\Theta)(\Theta\gamma_{c'}\gamma_{ab}\gamma^7\Theta) \\
&+ e_D^\mu (\Theta\Gamma^D\delta_\kappa\Theta) \left[e^c (\Theta\gamma_c\gamma_{ab}\gamma^5\gamma^7\Theta) + e^{c'} (\Theta\gamma_{c'}\gamma_{ab}\gamma^7\Theta) \right] \\
&- \frac{1}{2} dX^\mu (\Theta\gamma^c\gamma_{ab}\Theta)(\Theta\gamma_c\gamma^5\gamma^7\delta_\kappa\Theta) - dX^\mu (\delta_\kappa\Theta\gamma^{c'}\gamma^7\gamma_{ab}\Theta)(\chi\gamma_{c'}\gamma^5\theta) \\
&+ \frac{1}{2} dX^\mu (\Theta\gamma^{c'}\gamma^7\gamma_{ab}\Theta)(\theta\gamma_{c'}\gamma^5\delta_\kappa\theta + 2\chi\gamma_{c'}\gamma^5\delta_\kappa\theta) \\
&- \frac{1}{2} dX^\mu (\Theta\gamma^{c'}\gamma^7\gamma_{ab}\delta_\kappa\Theta)(\theta\gamma_{c'}\gamma^5\gamma_{ab}\theta + 2\chi\gamma_{c'}\gamma^5\gamma_{ab}\theta) \Big\} \\
&+ (a \to a', b \to b'), \tag{3.21}
\end{aligned}
$$

which are cubic order in the fermionic coordinate. After imposing the boundary condition of (3.5) as we did in previous paragraphs, we see that the constraints of (3.12) and the conditions (i) and (ii) suffice for showing that majority of terms vanish. However, the contributions involving ω^{ab} with $a \in N(D)$, $b \in D(N)$ and $\omega^{a'b'}$ with $a' \in N(D)$, $b' \in D(N)$ do not vanish. At this point, we would like to note that the spin connection for AdS$_4$ ($\mathbf{CP}^3$) has the schematic structure of $\omega^{ab} \sim X^{[a}dX^{b]}$ ($\omega^{a'b'} \sim X^{[a'}dX^{b']}$). This implies that the non-vanishing contributions vanish if the Dirichlet directions are set to zero. In other words, a given D-brane in the list of (3.13) except for $(2,1)$ and $(2,5)$ is 1/2-BPS if it is placed at the coordinate origin in its transverse directions.

Finally, we consider the terms in $S^{(4)}$ independent of the spin connection. In this case, it is enough to take the κ-symmetry variation only for Θ, since as seen from (3.1) $\delta_\kappa X^\mu$ leads to the contributions of higher order in Θ which should be treated with $\delta_\kappa S^{(6)}$. Then the boundary contributions from the κ-symmetry variation are read off as

$$
\begin{aligned}
\delta_\kappa S^{(4)} \longrightarrow \frac{R}{2k} \int_{\partial\Sigma} \Bigg\{ & \Big[-(\Theta\gamma^a\delta_\kappa\Theta)(\Theta\gamma_a\gamma^5\gamma^7 D\Theta) + (\Theta\gamma^a D\Theta)(\Theta\gamma_a\gamma^5\gamma^7\delta_\kappa\Theta) \\
& -(\theta\gamma^{a'}\gamma^5\delta_\kappa\theta + 2\chi\gamma^{a'}\gamma^5\delta_\kappa\theta)(\Theta\gamma_{a'}\gamma^7 D\Theta) \\
& +(\theta\gamma^{a'}\gamma^5 D_{24}\theta + 2\chi\gamma^{a'}\gamma^5 D_{24}\theta)(\Theta\gamma_{a'}\gamma^7\delta_\kappa\Theta) + 2(\chi\gamma^{a'}\gamma^5\theta)(\delta_\kappa\Theta\gamma_{a'}\gamma^7 D\Theta) \Big] \Big|_{\omega^{AB}=0} \\
& + \frac{i}{R}e^a \Big[2(\chi\gamma^5\chi)(\Theta\gamma_a\gamma^5\gamma^7\delta_\kappa\Theta) + 2(\chi\gamma^b\gamma^7\chi)(\Theta\gamma_{ab}\delta_\kappa\Theta) - 2(\chi\gamma_a\gamma^5\chi)(\Theta\gamma^5\gamma^7\delta_\kappa\Theta) \\
& \quad + 4(\delta_\kappa\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi)(\chi\gamma^{a'}\gamma^5\theta) - (\theta\gamma^b\delta_\kappa\theta + 2\chi\gamma^b\delta_\kappa\chi)(\chi\gamma_{ab}\gamma^7\chi) \\
& \quad - 2(\theta\gamma^{a'}\gamma^5\delta_\kappa\theta + 2\chi\gamma^{a'}\gamma^5\delta_\kappa\theta)(\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi) \Big] \\
& + \frac{i}{R}e^{a'} \Big[2(\chi\gamma^5\chi)(\Theta\gamma_{a'}\gamma^7\delta_\kappa\Theta) - 2(\chi\gamma^a\gamma^7\chi)(\Theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\delta_\kappa\Theta) - 2(\theta\gamma_{a'}\chi)(\Theta\gamma^5\gamma^7\delta_\kappa\Theta) \\
& \quad + 4(\delta_\kappa\Theta\gamma_{a'b'}\gamma^7\chi)(\chi\gamma^{b'}\gamma^5\theta) - 2(\theta\gamma^{b'}\gamma^5\delta_\kappa\theta + 2\chi\gamma^{b'}\gamma^5\delta_\kappa\theta)(\Theta\gamma_{a'b'}\gamma^7\chi) \\
& \quad + (\theta\gamma^a\delta_\kappa\theta + 2\chi\gamma^a\delta_\kappa\chi)(\theta\gamma_a\gamma_{a'}\gamma^5\gamma^7\chi) + \frac{1}{2}(\theta\gamma^{ab}\gamma^5\delta_\kappa\theta)(\theta\gamma_{ab}\gamma_{a'}\gamma^7\chi) \Big] \\
& - \frac{i}{6}e^a \left(\theta\gamma_a\gamma^5\gamma^7\mathcal{M}^2\delta_\kappa\theta - \delta_\kappa\theta\gamma_a\gamma^5\gamma^7\mathcal{M}^2\theta + \chi\gamma_a\gamma^5\gamma^7\mathcal{W}^2\delta_\kappa\chi \right) \\
& - \frac{i}{6}e^{a'} \left(\Theta\gamma_{a'}\gamma^7\mathcal{M}^2\delta_\kappa\theta - \delta_\kappa\Theta\gamma_{a'}\gamma^7\mathcal{M}^2\theta + \theta\gamma_{a'}\gamma^7\mathcal{W}^2\delta_\kappa\chi \right) \Bigg\}.
\end{aligned}
\tag{3.22}
$$

We see that there are lots of boundary contributions. One may wonder if all of them vanish without any extra condition after imposing the boundary condition (3.5) with the constraints (3.12) and the conditions (i) and (ii) below (3.18). However, lengthy but straightforward calculation indeed shows that the above boundary contibutions vanish without introducing any additional condition.

We have completed the investigation of the open string boundary condition for the κ-symmetry of the action expanded up to quartic order in Θ. The resulting classification of 1/2-BPS D-branes is summarized in table 1.

	D0	D2	D4	D6	D8
(n,n')	(1,0)	(0,3)	(1,4) (3,2)	(4,3)	(3,6)

Table 1. 1/2-BPS D-branes in the $\mathrm{AdS}_4 \times \mathbf{CP}^3$ background. n (n') represents the number of Neumann directions in AdS_4 ($\mathbf{CP}^3$). The Neumann directions in $\mathbf{CP}^3$ should follow the conditions (i) and (ii) below eq. (3.18). Each D-brane is supposed to have no worldvolume flux.

4 Discussion

We have given the covariant open string description of 1/2-BPS D-branes by investigating the suitable boundary condition which makes the boundary contributions from the κ-symmetry variation of the WZ term vanish up to the quartic order in Θ. As the main result, the 1/2-BPS D-branes in the $\mathrm{AdS}_4 \times \mathbf{CP}^3$ background have been classified as listed in table 1.

Although we do not have a rigorous proof, we expect that the classification is valid even at higher orders in Θ. In other words, any extra condition is expected to be unnecessary in showing the boundary κ-symmetry of the full WZ term. The reasoning behind this is due to the observation that the constraints of (3.12) for the possible 1/2-BPS D-bane configurations originate solely from the covariant derivative for Θ (2.10) incorporating the effects of background fields.[5] Note that the third term and the fourth term of (3.2) essentially comes from the variation of the first term of (2.3) involving the covariant derivative. This means that all the constraints are obtained just from the consideration of quadratic part $S^{(2)}$ (2.3). Of course, $S^{(2)}$ has the terms independent of the covariant derivative. However, if we trace the process of checking $\delta_\kappa S^{(2)}|_{\partial\Sigma} = 0$, we see that they lead to the vanishing boundary contributions consistently without requiring any additional constraint and have the boundary κ-symmetry. As we have checked in the previous section, for the quartic part $S^{(4)}$, the first non-trivial higher order part, again nontrivial contributions come from the quartic terms containing the covariant derivative. We expect that this situation continues to hold even for the higher order of Θ in the expansion of WZ term.

Actually, the above reasoning can be explicitly checked for the analogous open string descriptions of 1/2-BPS D-branes in some important supersymmetric backgrounds including Type IIA/IIB plane waves [18, 19] and $\mathrm{AdS}_5 \times \mathrm{S}^5$ [20–23] backgrounds. In all these cases, the quadratic part including the covariant derivative in the WZ term also determines the full classification of the 1/2-BPS D-branes. In particular, the result for the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background has been shown to be valid at full orders in the fermionic coordinate. That is, except from the quadratic part, we do not have any extra condition from higher order parts which might give further restriction on the 1/2-BPS D-brane configurations. For the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background, the string action can be obtained from the supercoset structure. Since $\mathrm{AdS}_4 \times \mathrm{S}^7$ has the similar supercoset structure and the $\mathrm{AdS}_4 \times \mathbf{CP}^3$ is obtained as an orbifold of $\mathrm{AdS}_4 \times \mathrm{S}^7$, we expect to prove the above reasoning explicitly, which will be an interesting topic to pursue.

---

[5]In order to describe 1/2-BPS D-branes, open string end points are placed at the coordinate origin of the Dirichlet directions. This eliminates the boundary contributions from the spin connection dependent terms.

One interesting fact about the $\text{AdS}_4 \times \mathbf{CP}^3$ background is that it is related to the Type IIA plane wave background through the Penrose limit [31]. The superstring action in the Type IIA plane wave background has been constructed in [32–34], and the open string description has been used to classify the 1/2-BPS D-branes in the background [19]. From the relation between two coordinate systems for the $\text{AdS}_4 \times \mathbf{CP}^3$ and the Type IIA plane wave backgrounds, we may compare the classfication data of table 1 with that obtained in [19]. Then we realize an agreement between them except for D0-brane. We note that, since non-trivial Kähler structure does not exist in the Type IIA plane wave background, the conditions below (3.18) due to the Kähler structure on $\mathbf{CP}^3$ disappear after taking the Penrose limit and hence two D-branes in (3.19) excluded from the 1/2-BPS D-branes turn out to be 1/2-BPS.

As for D0-brane, in contrast to the result in the $\text{AdS}_4 \times \mathbf{CP}^3$ background, it is not supersymmetric in the Type IIA plane wave background. The basic reason is simply the impossibility of taking a suitable open string boundary condition for D0-brane in a way of preserving supersymmetry. Given this discrepancy, one might wonder the fate of the supersymmetric D0-brane in the plane-wave limit. Starting from the usual AdS_4 metric

$$ds^2 = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh \rho^2 d\Omega_2^2 \tag{4.1}$$

we consider the boosted limit along an angle direction $\tilde{\psi}$ in $\mathbf{CP}^3$. Thus we define

$$x^+ = \frac{t + \tilde{\psi}}{2}, \ x^- = \tilde{R}^2 \frac{t - \tilde{\psi}}{2}. \tag{4.2}$$

Taking $\tilde{R} \to \infty$ limit with some additional scaling of other coordinates, we obtain the Type IIA plane-wave metric

$$ds^2 = -4dx^+ dx^- + \cdots . \tag{4.3}$$

The explicit construction was given at [31]. Note that in order to have the finite values of x^-,

$$t - \tilde{\psi} = o\left(\frac{1}{\tilde{R}^2}\right). \tag{4.4}$$

Thus the possible D0-brane configuration carried over to the plane-wave limit should satisfy eq. (4.4), which is necessarily nonsupersymmetric in $\text{AdS}_4 \times \mathbf{CP}^3$. In other words, the plane-wave limit is the geometry seen by the particle moving fast along the angle direction in $\mathbf{CP}^3$, D0-brane also should be comoving with that particle in order to have a sensible limit in the plane-wave geometry. We also would like to note that there is similar discrepancy between D1-branes in the $\text{AdS}_5 \times \text{S}^5$ and the type IIB plane-wave backgrounds also related through the Penrose limit [35]. As shown in [20–23], a Lorentzian D1-brane can be 1/2-BPS only when it is placed in the AdS_5 space. However, such D1-brane is not supersymmetric in the plane wave background and completely different type of configuration [18] appears to be supersymmetric which is furthermore not half but quarter BPS.

The classification of 1/2-BPS D-branes given in table 1 is 'primitive' in a sense that it gives no more information about 1/2-BPS D-branes. For example, it does not tell us about which configuration of a given D-brane is really 1/2-BPS and which part of the

background supersymmetry is preserved on the D-brane worldvolume. We should consider these questions by using other methods. One possible way would be to take the process adopted in [36, 37] for studying worldvolume theories on 1/2-BPS D-branes in the $AdS_5 \times S^5$ background. An important point we would like to note here is that it is enough to consider D-brane configurations based on the classification shown in table 1 . We do not need to investigate all possible configurations for the study of 1/2-BPS D-branes. Therefore, the classification provides us a good guideline or starting point for further exploration of the 1/2-BPS D-branes.

Acknowledgments

References

[1] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, $N = 6$ *superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [INSPIRE].

[2] D. Berenstein and D. Trancanelli, *Three-dimensional $N = 6$ SCFT's and their membrane dynamics*, *Phys. Rev.* **D 78** (2008) 106009 [arXiv:0808.2503] [INSPIRE].

[3] T. Nishioka and T. Takayanagi, *Fuzzy ring from M2-brane giant torus*, *JHEP* **10** (2008) 082 [arXiv:0808.2691] [INSPIRE].

[4] D. Berenstein and J. Park, *The BPS spectrum of monopole operators in ABJM: towards a field theory description of the giant torus*, *JHEP* **06** (2010) 073 [arXiv:0906.3817] [INSPIRE].

[5] A. Hamilton, J. Murugan, A. Prinsloo and M. Strydom, *A note on dual giant gravitons in $AdS_4 \times CP^3$*, *JHEP* **04** (2009) 132 [arXiv:0901.0009] [INSPIRE].

[6] M.M. Sheikh-Jabbari and J. Simon, *On half-BPS states of the ABJM theory*, *JHEP* **08** (2009) 073 [arXiv:0904.4605] [INSPIRE].

[7] M. Herrero, Y. Lozano and M. Picos, *Dielectric 5-branes and giant gravitons in ABJM*, *JHEP* **08** (2011) 132 [arXiv:1107.5475] [INSPIRE].

[8] D. Giovannoni, J. Murugan and A. Prinsloo, *The Giant graviton on $AdS_4 xCP^3$ - another step towards the emergence of geometry*, *JHEP* **12** (2011) 003 [arXiv:1108.3084] [INSPIRE].

[9] Y. Lozano, J. Murugan and A. Prinsloo, *A giant graviton genealogy*, *JHEP* **08** (2013) 109 [arXiv:1305.6932] [INSPIRE].

[10] C. Cardona and H. Nastase, *Open strings on D-branes from ABJM*, *JHEP* **06** (2015) 016 [arXiv:1407.1764] [INSPIRE].

[11] S. Hohenegger and I. Kirsch, *A note on the holography of Chern-Simons matter theories with flavour*, *JHEP* **04** (2009) 129 [arXiv:0903.1730] [INSPIRE].

[12] D. Gaiotto and D.L. Jafferis, *Notes on adding D6 branes wrapping Rp^3 in $AdS_4 \times CP^3$*, *JHEP* **11** (2012) 015 [arXiv:0903.2175] [INSPIRE].

[13] Y. Hikida, W. Li and T. Takayanagi, *ABJM with flavors and FQHE*, *JHEP* **07** (2009) 065 [arXiv:0903.2194] [INSPIRE].

[14] M. Ammon, J. Erdmenger, R. Meyer, A. O'Bannon and T. Wrase, *Adding flavor to AdS_4/CFT_3*, *JHEP* **11** (2009) 125 [arXiv:0909.3845] [INSPIRE].

[15] B. Chandrasekhar and B. Panda, *Brane Embeddings in $AdS_4 \times CP^3$*, *Int. J. Mod. Phys.* **A 26** (2011) 2377 [arXiv:0909.3061] [INSPIRE].

[16] H. Kim and N. Kim, *On the supersymmetry of spiky branes in $AdS_4 \times CP^3$*, *Mod. Phys. Lett.* **A 25** (2010) 3267 [INSPIRE].

[17] N.D. Lambert and P.C. West, *D-branes in the Green-Schwarz formalism*, *Phys. Lett.* **B 459** (1999) 515 [hep-th/9905031] [INSPIRE].

[18] P. Bain, K. Peeters and M. Zamaklar, *D-branes in a plane wave from covariant open strings*, *Phys. Rev.* **D 67** (2003) 066001 [hep-th/0208038] [INSPIRE].

[19] S.-j. Hyun, J. Park and H.-j. Shin, *Covariant description of D-branes in IIA plane wave background*, *Phys. Lett.* **B 559** (2003) 80 [hep-th/0212343] [INSPIRE].

[20] M. Sakaguchi and K. Yoshida, *D-branes of covariant AdS superstrings*, *Nucl. Phys.* **B 684** (2004) 100 [hep-th/0310228] [INSPIRE].

[21] M. Sakaguchi and K. Yoshida, *Notes on D-branes of type IIB string on $AdS_5 \times S^5$*, *Phys. Lett.* **B 591** (2004) 318 [hep-th/0403243] [INSPIRE].

[22] E. Chang-Young, H. Nakajima and H. Shin, *Consistent bilinear Wess-Zumino term for open AdS superstring*, *JHEP* **10** (2012) 048 [arXiv:1207.7210] [INSPIRE].

[23] S. Hanazawa and M. Sakaguchi, *D-branes from pure spinor superstring in $AdS_5 \times S^5$ background*, *Nucl. Phys.* **B 914** (2017) 234 [arXiv:1609.05457] [INSPIRE].

[24] G. Arutyunov and S. Frolov, *Superstrings on $AdS_4 \times CP^3$ as a coset σ-model*, *JHEP* **09** (2008) 129 [arXiv:0806.4940] [INSPIRE].

[25] B. Stefanski, jr, *Green-Schwarz action for type IIA strings on $AdS_4 \times CP^3$*, *Nucl. Phys.* **B 808** (2009) 80 [arXiv:0806.4948] [INSPIRE].

[26] P. Fré and P.A. Grassi, *Pure spinor formalism for $Osp(N|4)$ backgrounds*, *Int. J. Mod. Phys.* **A 27** (2012) 1250185 [arXiv:0807.0044] [INSPIRE].

[27] R. D'Auria, P. Fré, P.A. Grassi and M. Trigiante, *Superstrings on $AdS_4 \times CP^3$ from supergravity*, *Phys. Rev.* **D 79** (2009) 086001 [arXiv:0808.1282] [INSPIRE].

[28] J. Gomis, D. Sorokin and L. Wulff, *The complete $AdS_4 \times CP^3$ superspace for the type IIA superstring and D-branes*, *JHEP* **03** (2009) 015 [arXiv:0811.1566] [INSPIRE].

[29] P.A. Grassi, D. Sorokin and L. Wulff, *Simplifying superstring and D-brane actions in $AdS_4 \times CP^3$ superbackground*, *JHEP* **08** (2009) 060 [arXiv:0903.5407] [INSPIRE].

[30] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, *Class. Quant. Grav.* **1** (1984) 499 [INSPIRE].

[31] T. Nishioka and T. Takayanagi, *On type IIA Penrose limit and $N = 6$ Chern-Simons theories*, *JHEP* **08** (2008) 001 [arXiv:0806.3391] [INSPIRE].

[32] K. Sugiyama and K. Yoshida, *Type IIA string and matrix string on PP wave*, *Nucl. Phys.* **B 644** (2002) 128 [hep-th/0208029] [INSPIRE].

[33] S.-j. Hyun and H.-j. Shin, $N = (4, 4)$ *type* $2A$ *string theory on PP wave background*, *JHEP* **10** (2002) 070 [hep-th/0208074] [INSPIRE].

[34] J. Park and H. Shin, *Complete type IIA superstring action on IIA plane wave background*, *JHEP* **06** (2012) 111 [arXiv:1203.6104] [INSPIRE].

[35] M. Blau, J.M. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. Quant. Grav.* **19** (2002) L87 [hep-th/0201081] [INSPIRE].

[36] J. Park and H. Shin, *Notes on worldvolume supersymmetries for D-branes on* $AdS_5 \times S^5$ *background*, *JHEP* **09** (2017) 022 [arXiv:1705.06887] [INSPIRE].

[37] S.K. Domokos and A.B. Royston, *Holography for field theory solitons*, *JHEP* **07** (2017) 065 [arXiv:1706.00425] [INSPIRE].